

Changes in the Duration and Peak of the Wave Season on Puerto Rico

By

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Start: Jan-5-2016, Finished: Jan-30-2016, Modified: May-7-2016 (included NAO index, Fig. 8B)

Abstract

Empirical Mode Decomposition (EMD) analysis was performed on wave height data recorded between years 2010-2015 by NDBC and CariCOOS wave buoys located north of Puerto Rico. The analysis revealed that the eleventh intrinsic mode function (IMF) named C11 shows an annual time scale; this physical signal corresponds to the wave season on Puerto Rico. The peak and period between zero-crossings in C11 corresponds to the peak and duration of the wave season, respectively. The peak's position can shift from late January to early November. The duration of the wave season ranged between 5 to 8 months. The shift of the peak towards the month of November increased the wave activity during the month of November when the amplitude of the annual tidal constituent is larger and mean sea level is higher. These conditions were favorable for coastal erosion during the 2010-2011 wave season. Our results revealed the true exact form and variability of the wave season on Puerto Rico. The gradual increase of the annual wave heights in combination with the gradual increase in the annual tidal water levels could have an important effect on coastal erosion in Puerto Rico.

Introduction

Five years of ocean wave data were recorded by two NDBC stations and one CariCOOS buoy located in the West Atlantic Ocean north of Puerto Rico. The ocean wave height time series is a nonlinear and nonstationary signal. Each wave height time series was subjected to the Empirical Mode Decomposition (EMD), a proven method to analyze nonstationary and nonlinear signals (Huang et al. 1998). This adaptive method has been applied to the analysis of nonlinear water waves (Huang et al. 1999). From EMD we obtain intrinsic mode functions (IMF) components that have a physical interpretation. Well known nonlinear systems such as the Duffing equation has been subject to EMD analysis and each component have a physical meaning. The goal of our analysis was to obtain the annual wave height signal at each station; this signal contained in one IMF corresponds to the wave season. The IMF is a stationary signal and can be subject to a Fourier Model fit. In theory with a sufficient long time series it is possible to generate a forecast of the next year wave season.

Identifying changes in the wave season, such as the duration (start to end) and the date of the peak amplitude are crucial to understand the impact of that change on beach processes, such as erosion (or

accretion). Another important aspect is that under climate change we expect changes in the wave climate (see References). We need to track this long term changes in the annual component of the wave height time series. A careful monitoring of the IMF that contains this annual component can provide key information about the changes in wave climate.

Methodology

Data Collection

For this study we selected historic wave heights measured every hour at National Data Buoy Center (NDBC) stations 41046, 41043; and Caribbean Coastal Ocean Observing System (CariCOOS) PR-2 station. For the specific location of the stations please visit the [NDBC](#) and [CariCOOS](#) websites. The data was downloaded from the NDBC website and CariCOOS website. We avoided the use of data with gaps larger than two weeks. Data with gaps of the order of one-month are too wide for our EMD analysis. The CariCOOS PR-2 time series was truncated because after July 1st 2014 there is data with a gap larger than one month. Below we display the time span of each time series:

Station Identification	Start Series	End Series	Number of Days
NDBC Station 41046	31-Dec-2009 23:51:28	31-May-2015 22:53:10	1977
NDBC Station 41043	08-May-2010 09:50:24	30-Nov-2015 22:48:00	2032.5
CariCOOS Buoy PR-2	23-Jul-2010 15:00:00	01-Jul-2014 06:00:00	1438.6
CO-OPS Station 9755371	01-Jan-2010 00:00:00	30-Nov-2015 23:02:24	2160.0

In addition, water level records were obtained from the San Juan Harbor tide gauge station (NOAA/NOS/CO-OPS/Station 9755371).

Empirical Mode Decomposition (EMD)

Empirical Mode Decomposition (EMD) and Hilbert Spectrum Analysis (HSA) are the two main components of Hilbert Huang Transform (HHT) analysis. HHT performs time-frequency analysis of nonstationary signals, as do other techniques such as wavelets. However, HHT is better suited than wavelets for analyzing signals resulting from nonlinear processes (Huang et al. 1998). A detailed explanation can be found in Huang et al. (1998, 1999). EMD can separate the signal into orthogonal components with different time scales. The successful application of EMD for decomposing nonstationary and nonlinear oceanic physical phenomena in Puerto Rico has been reported in a number of papers (Huang et al. 2000, Alfonso-Sosa, 2010; 2009). That previous experience motivated us to choose EMD to analyze our wave height data. This technique decomposes time series data into a finite number of **intrinsic mode functions (IMFs)** with time variable amplitudes and frequencies. The decomposition is orthogonal and adaptive. By adaptive we mean that the EMD decomposition adapts to the local variations of the data. Adaptive basis is indispensable for nonstationary and nonlinear data analysis. Any function is an IMF if (1) in the whole data set, the number of extrema and the number of zero-crossings is either equal or differ at most by one, and (2) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima are zero. The

second criterion means that the function has symmetric envelopes defined by local maxima and minima respectively. The process to achieve this decomposition is called “extrema sifting”. Maximum sift times was 10. Each wave height time series was subjected to the Empirical Mode Decomposition (EMD). The original time series was decomposed into fourteen IMFs and one residual. After the wave height time series is decomposed into IMFs we can easily identify the annual signal from other time scales (Figure 1). We focus on the eleventh IMF, named C11, which represents the annual component of the wave height time series.

This analysis was performed in Matlab, using the m-files provided by: 中央大學數據分析中 Research Center for Adaptive Data Analysis. Chungli,Taiwan <http://rcada.ncu.edu.tw/intro.html>

The same procedure was applied to the water level records (2010-2015) obtained from the San Juan Harbor tide gauge station (NOAA/NOS/CO-OPS/Station 9755371).

C11 is a stationary time series with an annual oscillation scale and a Fourier series model easily adjusts to its data. The IMF C11 was least-squared fitted with a Fourier series model.

Fourier series Model

The Fourier series is a sum of sine and cosine functions that describes a periodic signal. It is represented in trigonometric form

$$y = a_0 + \sum_{i=1}^n a_i \cos(iwx) + b_i \sin(iwx)$$

where a_0 models a constant (intercept) term in the data and is associated with the $i = 0$ cosine term, w is the fundamental frequency of the signal, n is the number of terms (harmonics) in the series, and $1 \leq n \leq 8$.

For our particular case, C11 was fitted with the first term of the Fourier model. Despite adding more terms (up to eight) they did not increase significantly the fit. The reason for this is that C11 is a stationary one-scale signal and one term in the Fourier model is enough to model most of its variability.

Results

Figure 1 display fifteen intrinsic mode functions (IMF) obtained from EMD analysis of historic wave heights measured every hour at NDBC station 41043 between May 2010 and November 2015. The EMD analysis of the 5-year wave height time series reveals an annual scale in the eleventh IMF, C11. C11 shows five distinct peaks with amplitudes of less than 0.5 meters, each one around the month of January. The valleys occur around mid-year. This pattern corresponds to the wave season in Puerto Rico. According to the NWS in San Juan the swell season, climatology speaking, extends from November 1st through March (Anselmi-Molina, 2015). The first six IMFs, C1-C6, contain amplitudes larger than 0.5 meters and individual big wave events can be easily identified in each of them. The plot shows that C11 shifted its peak toward November when the bigger wave events tend to occur before January 1st, like in year 2010. On the contrary, C11 shifted its peak toward February when the bigger wave events tend to

occur after January 1st, like in year 2014. This behavior of C11 convinced us that it represents a real physical signal and corresponds to the wave season. Figure 2 show the original series and IMF C11 for each of the stations: NDBC 41046, NDBC 41043 and CariCOOS PR-2. It is clear to see from Figure 2 that the positive values of C11 are confined inside the period between October and April when the wave heights in the original series are generally above the 2 meters threshold (see middle panel). The time span or duration between the ascending and descending zero-crossings of C11 corresponds to the period of bigger waves in the three stations.

Figure 3 shows the superposition of IMF C11 for each one of the three stations. Peak amplitudes values ranged between 0.29 and 0.5 meters. The largest peak amplitudes are in the northernmost station NDBC 41046 and the smaller ones in CariCOOS PR-2. Smaller peak amplitudes were observed in wave season 2013-2014. Larger peak amplitudes were observed in wave seasons 2010-2011 and 2011-2012. The peak amplitudes do not occur at the exact time in any station. The difference can be as large as 58 days. The peaks of C11 for the station 41043 and PR-2 on November 2010 and on January 2012, respectively, differed by less than two weeks (see Table-1). On January 2014 the peaks at station 41043 and station 41046 differed only by 18 days. Figure 3 shows that the separations between zero crossings are not equal; some years the duration between an ascending and descending zero crossings can be as short as 5 months and as long as 8 months (see Table-1). In particular for the wave season 2012-13 the duration between zero-crossings was longer, starting early October and ending on May. The ascending zero crossing in station 41046 occurred on September in three wave seasons: 2011-12, 2012-13, and 2014-15. On these wave seasons the duration was longer than for the other stations (Figure 4B). The average durations for stations 41046, 41043 and PR-2 are 6.95, 6.38 and 6.05 months, respectively. By establishing January 1st as an origin we can calculate the displacement of the peaks at each station (see Figure 4A). Station 41046 shows on three wave seasons (2010-11, 2011-12, and 2012-13) a peak before January 1st. Only for the wave season 2010-11 the peaks occurred before January 1st in the three stations. Most of the peaks occurred less than 30 days from January 1st.

Figure 5 shows a superposition of IMF C11 of CariCOOS-PR2, the Mean Sea Level (MSL) at station 9755371 and IMF C10 obtained from the EMD analysis of the water levels at station CO-OPS/9755371. IMF C10 represents the annual signal of the water levels at San Juan Harbor. Figure 5 shows that the maximum annual tidal water levels and MSL usually occur near October 1st. IMF C10 peaks occurred on the following dates: 01-NOV-2010, 8-SEP-2011, 16-SEP-2012, 4-OCT-2013, and 5-OCT-2014. The maximum water levels were: 0.1329 m, 0.1447 m, 0.1464 m, 0.1161 m, and 0.1665 m. The peaks of IMF C11 usually do not coincide with the maximum annual tidal water levels (IMF C10) because during the peak of the wave season the annual tidal levels are decreasing. It is clear that on most of the years the peak of the annual wave heights did not coincided exactly with the peaks of the annual tidal level. But an exception to this happen when the peak of IMF C11 shifted toward November 5th 2010, about 4 days from November 1st (peak of IMF C10), larger wave heights and higher tidal levels combined. The peak of wave height on November 5th was 0.2893 m and the peak in tidal level was 0.1329 m, its sum is 0.4222 m.

IMF C11 is one time scale function and due to its stationary nature it is possible to adjust it a one-term Fourier model. This model fit can be extrapolated to forecast for the next wave season if the goodness

of fit statistics are good ($R\text{-square} > 0.95$). Fourier series least-square fit of the annual component (C11) for all the stations are shown in Figure 6A-6C and their fit-statistics are shown in the next page. The best goodness of fit statistics is for station 41046, with an $R\text{-square}$ of 0.87. Figure 7 shows a superposition of all the fits. Most of the peaks of the fits occurred between late December and early January of the next year. No more than two weeks before or after January 1st.

Figure 9 shows the occurrences of big wave events between October and March with heights larger than 4 meters. Figure 8A shows the position of C11's peak of wave heights. C11's peak position shifts to a date before January 1st during wave seasons: 2010-2011 and 2012-2013. On these same seasons most of the big wave events occurred before January 1st. On the contrary, C11's peak position shifts to a date after January 1st during wave seasons: 2011-2012 and 2013-2014. Again on these seasons most of the big wave events occurred after January 1st. Wave season 2014-2015 did not follow that pattern. Figure 8B shows the monthly North Atlantic Oscillation (NAO) index and when we compared it with Figure 8A revealed that during previous months and into the first half (Oct-Dec) of wave seasons: 2010-2011 and 2012-2013 the NAO index values were consistently negative. On the contrary, positive index values during the whole (Oct-Apr) wave seasons: 2011-12, 2013-14 and 2014-15 correspond to peaks in wave activity occurring after January 1st.

Discussion

The above results demonstrated that IMF C11 is a real physical signal corresponding to the wave season on Puerto Rico. IMFs C1 to C6 contain the individual big wave events and all the high frequency part of the original signal, but C11 contains only the annual wave height variation (Figure 9). This low frequency signal is almost independent of the other IMFs because they are almost numerically orthogonal although not exactly orthogonal. IMF C11 represents the gradual increase (or decrease) of wave heights in the Atlantic Ocean and does not include any big wave events. The shift of C11's peak and changes in the time span between zero-crossings revealed the annual variability of Puerto Rico's wave seasons. Some wave seasons showed peaks in wave height up to 2 months before January 1st and other 30 days after that date. The time span between zero crossings in C11 revealed that the duration of the wave season runs between 5 to 8 months. From the Fourier model fit we conclude that the peak of a typical wave season it is expected to occur in less than 15 days after January 1st. A two month shift of the peak into November can put larger than usual wave activity dangerously near the time of the highest tides of the year. This untypical condition is favorable for an increase in coastal erosion along the north coast of Puerto Rico. Major changes in beach profile were found by UPR scientists occurring during 2010 at selected beaches on the north coast of Puerto Rico (Barreto and Cabrera, 2014). It is very interesting the idea that the gradual increase of the annual wave heights signal in combination with the gradual increase in the annual tidal water levels signal could have an important effect on coastal erosion. This relationship is very subtle and inconspicuous for the majority of us. It can only be detected by a careful analysis of both signals. In contrast, the big wave events are conspicuous, and obviously reported in the news media. Usually we focus our attention in the big swell events as the major actors of the coastal erosion during the fall and winter. Future studies must pay more attention to the wave season itself (defined by IMF C11) to understand its role in coastal erosion. Our results showed that on those seasons

with earlier (later) peaks there is a tendency to earlier (later) big wave events ($H > 4$ m). More data it's necessary to determine the exact correspondence (if any) between the annual wave season and any separate big wave events.

Our results revealed that during the negative phase of the NAO the peak of wave activity was shifted to earlier in the season. During this negative phase the location of the North Atlantic jet stream and storm track moves southward and the East Coast of the United States exhibits colder weather. These conditions are favorable for the generation of cold fronts and northeasters that increase the wave climate offshore the East Coast. In a couple of days these waves travel toward Puerto Rico and increase our wave regime.

Additional data is necessary for a better Fourier model fit ($R\text{-square} > 0.95$) necessary for forecasting. The short data record (5 years) is not enough for an accurate forecasting to the next wave season.

Conclusion

EMD analysis revealed the annual component of the wave height time series. This IMF with an annual time scale represents the wave season on Puerto Rico. The peak and duration of the wave season changes every year. The peak of the wave season can shift between one and two months from its typical or expected position. The duration of the wave season is variable and can be as short as 5 months and as long as 8 months. By limiting the boundaries of the wave season to specific starting and ending dates does not reflect the variability of a real wave season. It is more appropriate a dynamic view of the wave season instead of a static one. Wave season 2010-2011 was untypical because the annual peak of wave heights coincided with the annual peak of the tidal heights. This could explain the higher rates of erosion observed in the 2010-2011 wave season. An earlier peak of the wave season should be expected when the NAO index shows negative values previous and during the first half of the wave season (Oct-Dec).

Acknowledgements

We thank CariCOOS, NOAA/NDBC, and NOAA/NOS/CO-OPS for all the data used in this work. Their respective websites are shown below:

NOAA/NDBC Website: <http://www.ndbc.noaa.gov/>

NOAA/NOS/CO-OPS Web site: <http://tidesandcurrents.noaa.gov/>

CariCOOS Website: <http://www.caricoos.org/>

North Atlantic Oscillation website: <http://www.ncdc.noaa.gov/teleconnections/nao/>

Figure Captions

Figure 1. Fifteen IMF components obtained from EMD analysis of historic wave heights measured every hour at NDBC station 41043 between April 2010 and November 2015. The eleventh IMF named C11 is the annual component of the wave heights. The y-axis limits of C11 are [-0.5 m 0.5 m]. The last plot is the original signal before EMD analysis. Wave heights are expressed in meters.

Figure 2A-2C. Wave heights recorded at two NDBC stations and one CariCOOS station with their corresponding eleventh IMF, named C11. Top. NDBC station 41046. Middle. NDBC station 41043. Bottom. CariCOOS Buoy PR-2. The green horizontal line represents the two meters wave height threshold. The annual component, C11, show gaps for stations 41046 and PR-2.

Figure 3. Superposition of the three annual constituents (C11) for the following stations: NDBC 41046 (dotted line, light green), NDBC 41043 (black line) and CariCOOS PR-2 (dashed line, dark green). The table below shows the date for each peak of the annual component (C11) and its zero-crossings.

Figure 4A-4B. Top. Number of days from January 1st to the peak of the annual component (C11). Negative (positive) days indicate before (after) January 1st. Bottom. Number of days between the ascending and descending zero-crossings of the annual component (C11).

Figure 5. Plot of the annual component (IMF C11) of wave heights measured at CariCOOS PR-2 buoy (dashed line, dark green), the annual component (IMF 10) of water levels measured at NOAA/NOS/CO-OPS tidal station 9755371 (black line) and the mean sea-level (MSL) recorded at the same station (dot-line, green).

Figure 6A-6C. Fourier series least-square fit of the annual component (C11) for the following stations: Top. NDBC station 41046. Middle. NDBC station 41043. Bottom. CariCOOS Buoy PR-2. Fit statistics are shown in the next page.

Figure 7. This plot shows a superposition of the fits shown in Figure 6A-6C. The red line is station NDBC 41043, dotted line corresponds to station NDBC 41046, and dash-dot line corresponds to PR-2. Most of the peaks in wave height run between the 28th December and the 14th of January. The table below shows the date for each peak of the fit.

Figure 8A-8B. Top. Number of days from January 1st to the peak of the annual component (C11). Negative (positive) days indicate before (after) January 1st. The data is from station NDBC 41043. Bottom. Monthly values of the North Atlantic Oscillation (NAO) index from 1/1/2010 to 12/1/2015. (NOAA/NCDC).

Figure 9. Number of days from January 1st to the day of an event with wave heights larger than 4 meters. Negative (positive) days indicate before (after) January 1st.

Figure 10. The sum of IMFs C1 to C6 is shown in blue and represents the high frequency part of the wave height time series, including big wave events. The black line is IMF C11 and represents the wave season.

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NOAA/NDBC Website: <http://www.ndbc.noaa.gov/>

NOAA/NOS/CO-OPS Web site: <http://tidesandcurrents.noaa.gov/>

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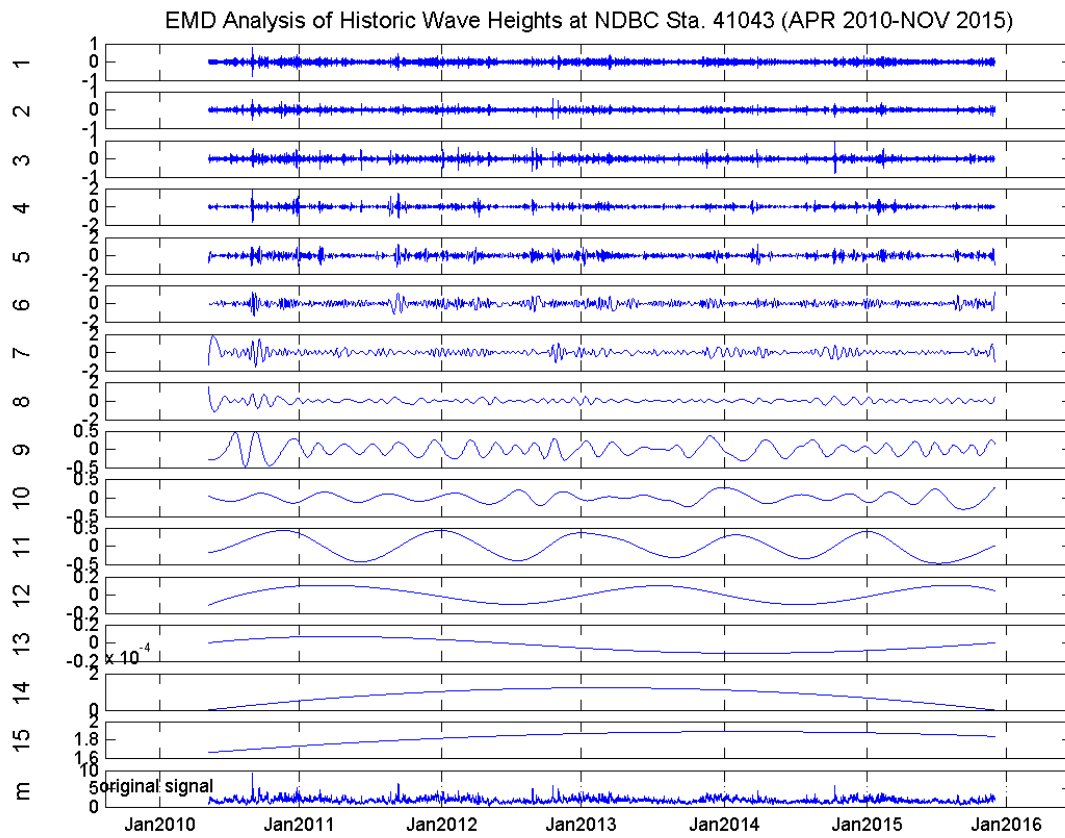


Figure 1

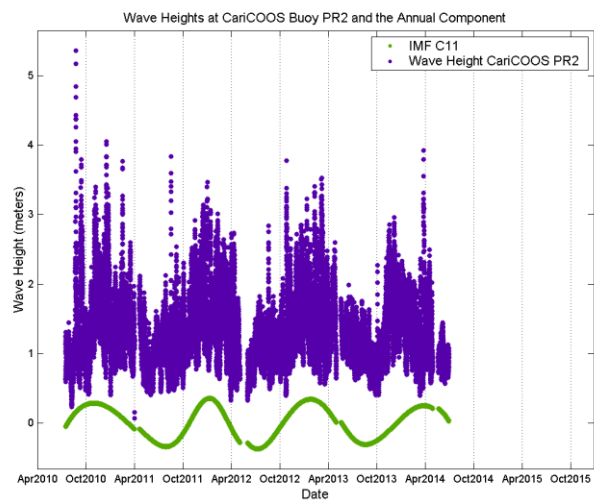
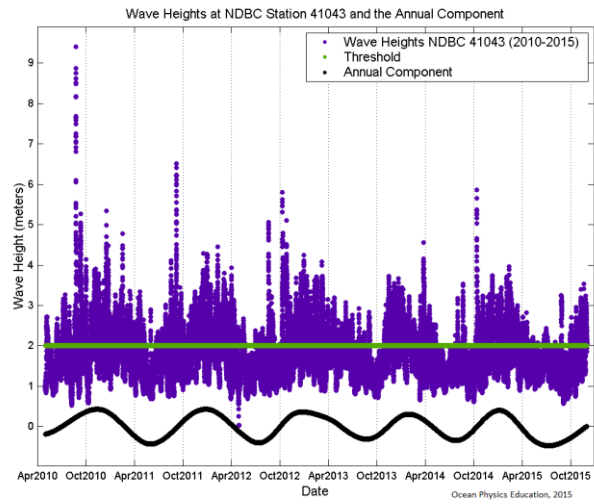
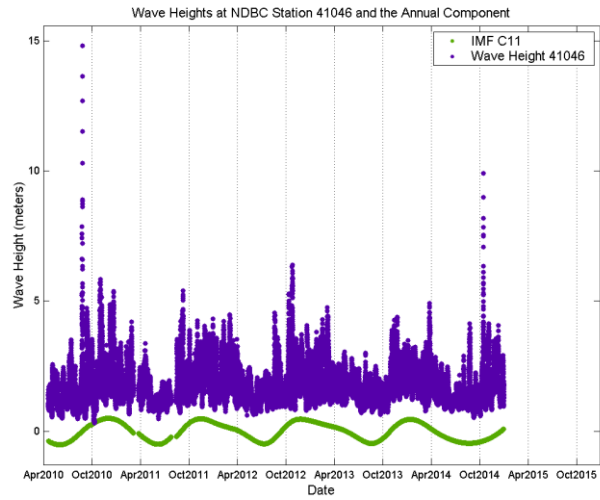


Figure 2A-2C

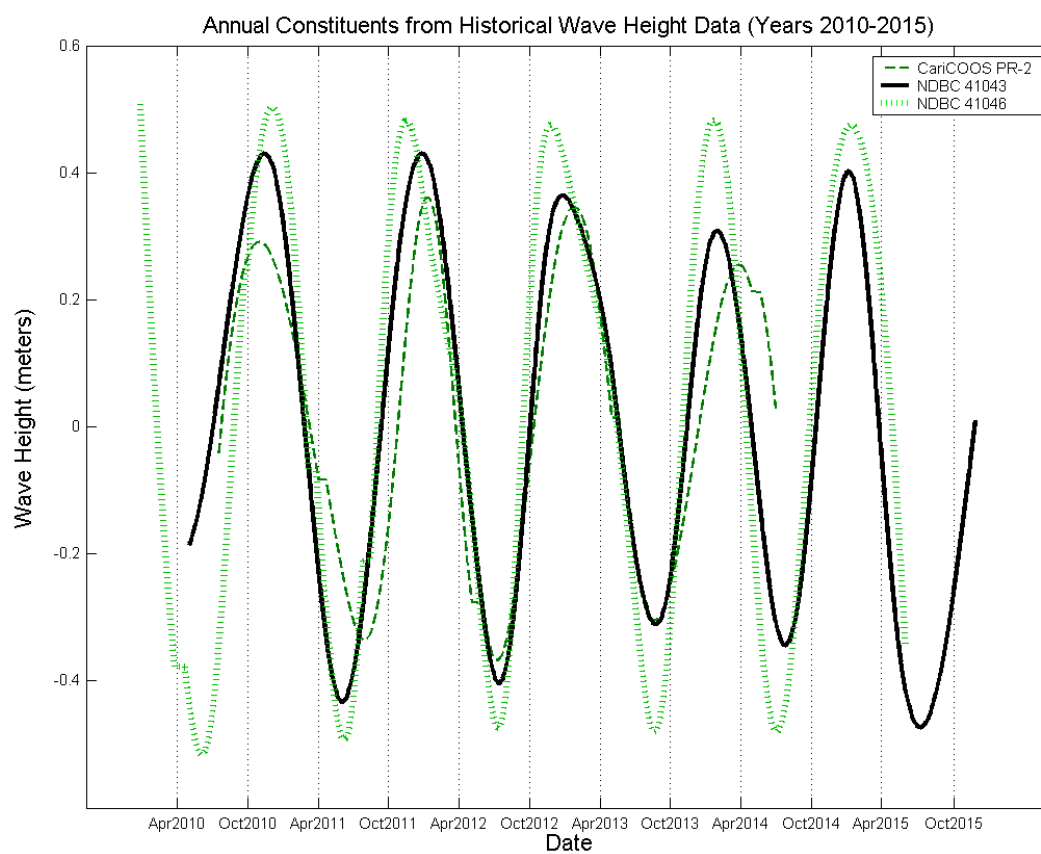


Figure 3

Table 1. Dates of peaks and zero-crossings of the Annual Component IMF C11. Duration between ascending and descending zero-crossings was expressed in months.

Dates of Maximum Height of the Annual Component IMF C11		
NDBC 41046	NDBC 41043	CariCOOS PR2
8-Dec-10	18-Nov-10	5-Nov-10
19-Nov-11	4-Jan-12	14-Jan-12
27-Nov-12	26-Dec-12	24-Jan-13
13-Jan-14	31-Jan-14	
	6-Jan-15	

Dates of Zero Crossings of IMF C11					
NDBC 41046			NDBC 41043		
Ascending	Descending	Duration	Ascending	Descending	Duration
	5-Mar-11			1-Mar-11	
7-Sep-11	13-Apr-12	7.3	16-Sep-11	13-Apr-12	7.0
16-Sep-12	16-May-13	8.1	5-Oct-12	21-May-13	7.6
5-Nov-13	14-Apr-14	5.3	15-Nov-13	30-Apr-14	5.5
27-Sep-14	29-Apr-15	7.1	14-Oct-14	26-Mar-15	5.4

CariCOOS PR2		
Ascending	Descending	Duration
	15-Mar-11	
29-Oct-11	26-Mar-12	5.0
15-Oct-12	17-May-13	7.1
16-Dec-13		

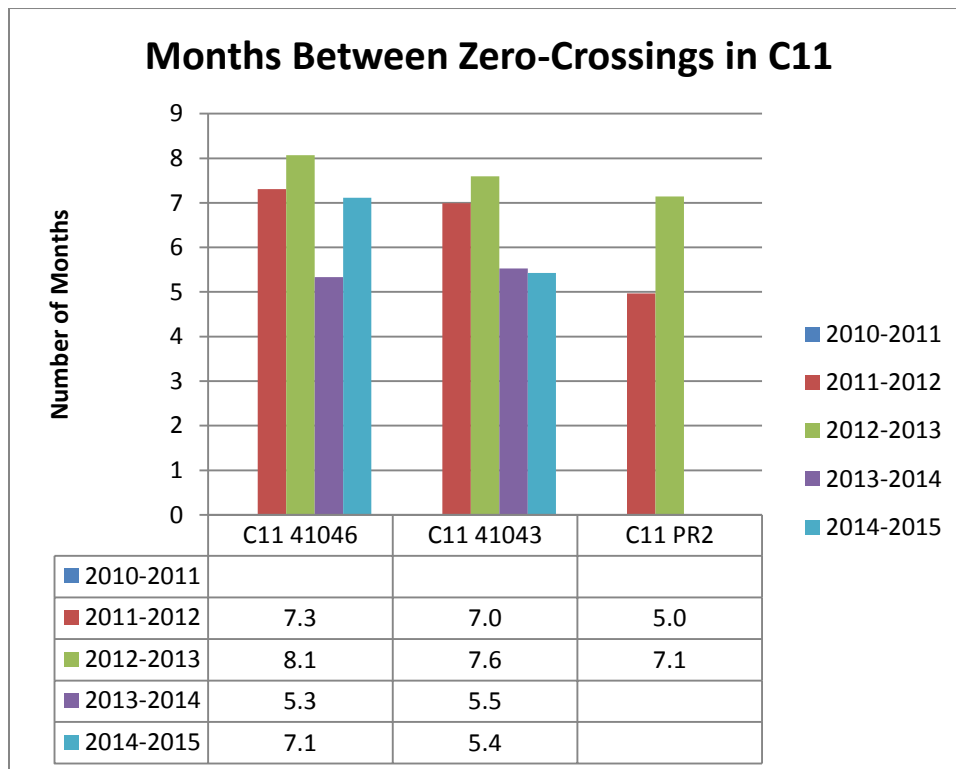
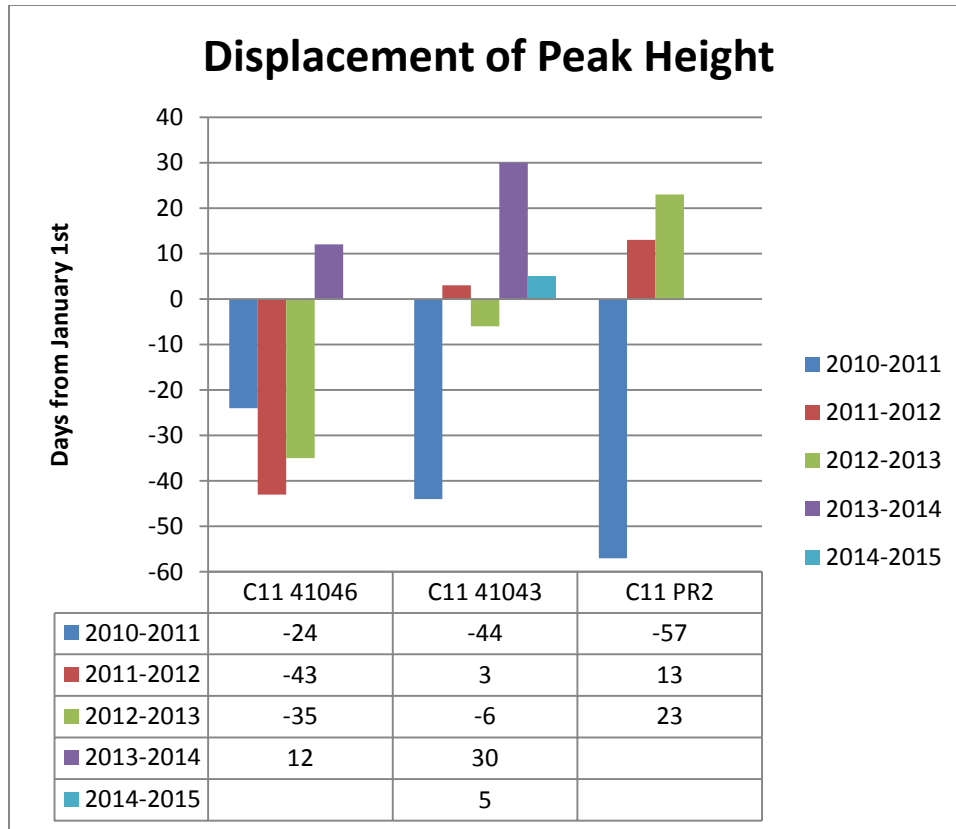
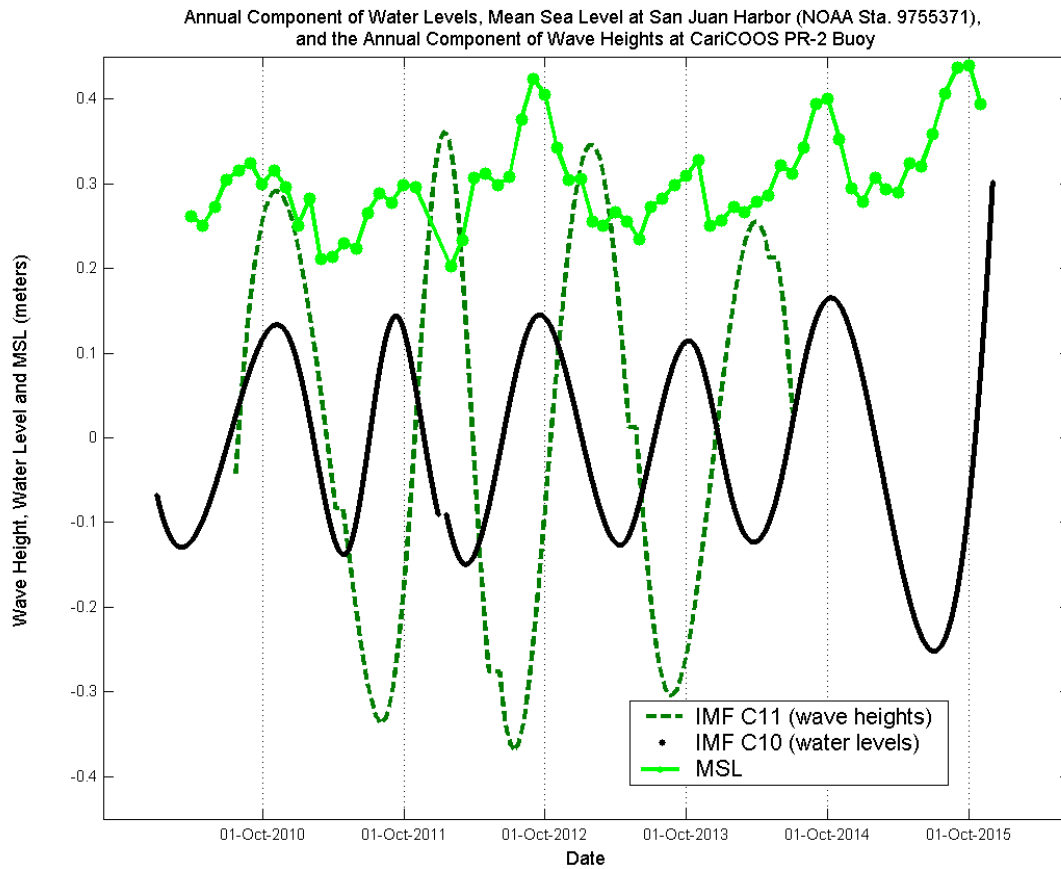


Figure 4A-4B



Dates of Peak of the Annual Tidal Component of Water Levels in San Juan Harbor (IMF C10)

1-Nov-2010
8-Sep-2011
16-Sep-2012
4-Oct-2013
5-Oct-2014

Figure 5

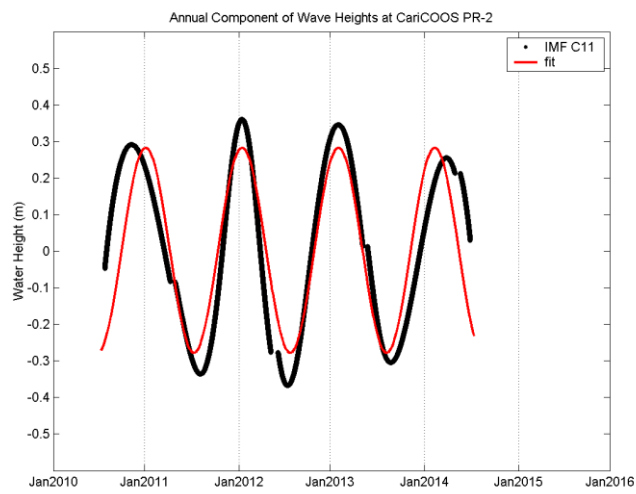
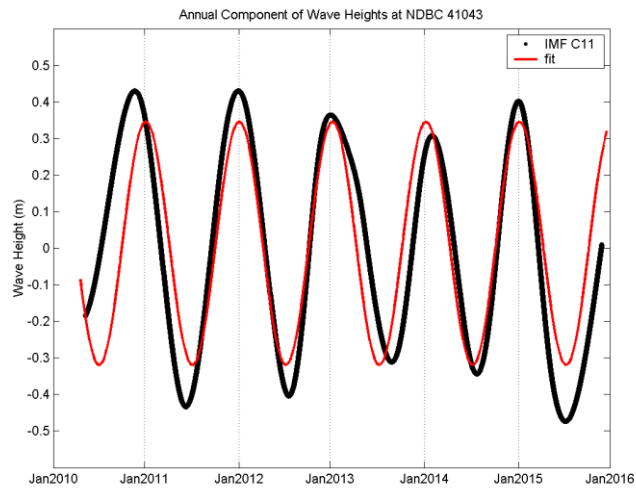
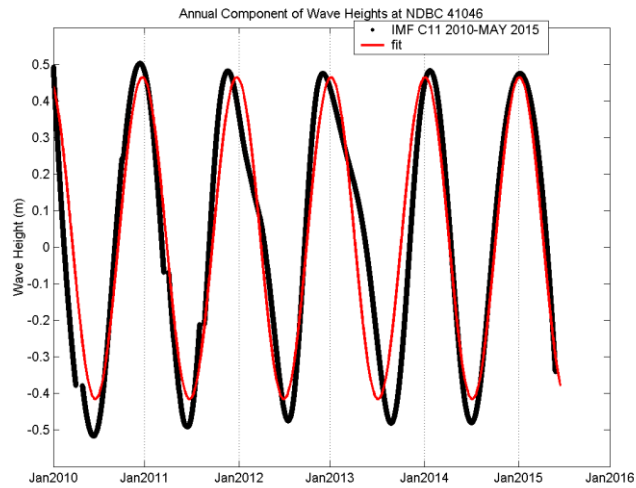


Figure 6A-6C.

NDBC 41046

General model Fourier1:

$$f(x) = a_0 + a_1 \cos(x \cdot w) + b_1 \sin(x \cdot w)$$

Coefficients (with 95% confidence bounds):

$$a_0 = 0.02435 \ (0.02323, 0.02548)$$

$$a_1 = -0.3948 \ (-1.288, 0.4984)$$

$$b_1 = 0.1937 \ (-1.627, 2.015)$$

$$w = 0.01694 \ (0.01694, 0.01695)$$

Goodness of fit:

SSE: 663.5

R-square: 0.8672

Adjusted R-square: 0.8672

RMSE: 0.1216

NDBC 41043

General model Fourier1:

$$f(x) = a_0 + a_1 \cos(x \cdot w) + b_1 \sin(x \cdot w)$$

Coefficients (with 95% confidence bounds):

$$a_0 = 0.01291 \ (0.01158, 0.01424)$$

$$a_1 = 0.3245 \ (-0.1795, 0.8284)$$

$$b_1 = 0.07135 \ (-2.221, 2.363)$$

$$w = 0.01709 \ (0.01708, 0.0171)$$

Goodness of fit:

SSE: 1065

R-square: 0.7137

Adjusted R-square: 0.7137

RMSE: 0.1486

CariCOOS PR-2

General model Fourier1:

$$f(x) = a_0 + a_1 \cos(x \cdot w) + b_1 \sin(x \cdot w)$$

Coefficients (with 95% confidence bounds):

$$a_0 = 0.001948 \quad (0.000695, 0.0032)$$

$$a_1 = -0.04089 \quad (-3.033, 2.951)$$

$$b_1 = -0.277 \quad (-0.7185, 0.1644)$$

$$w = 0.01657 \quad (0.01655, 0.01658)$$

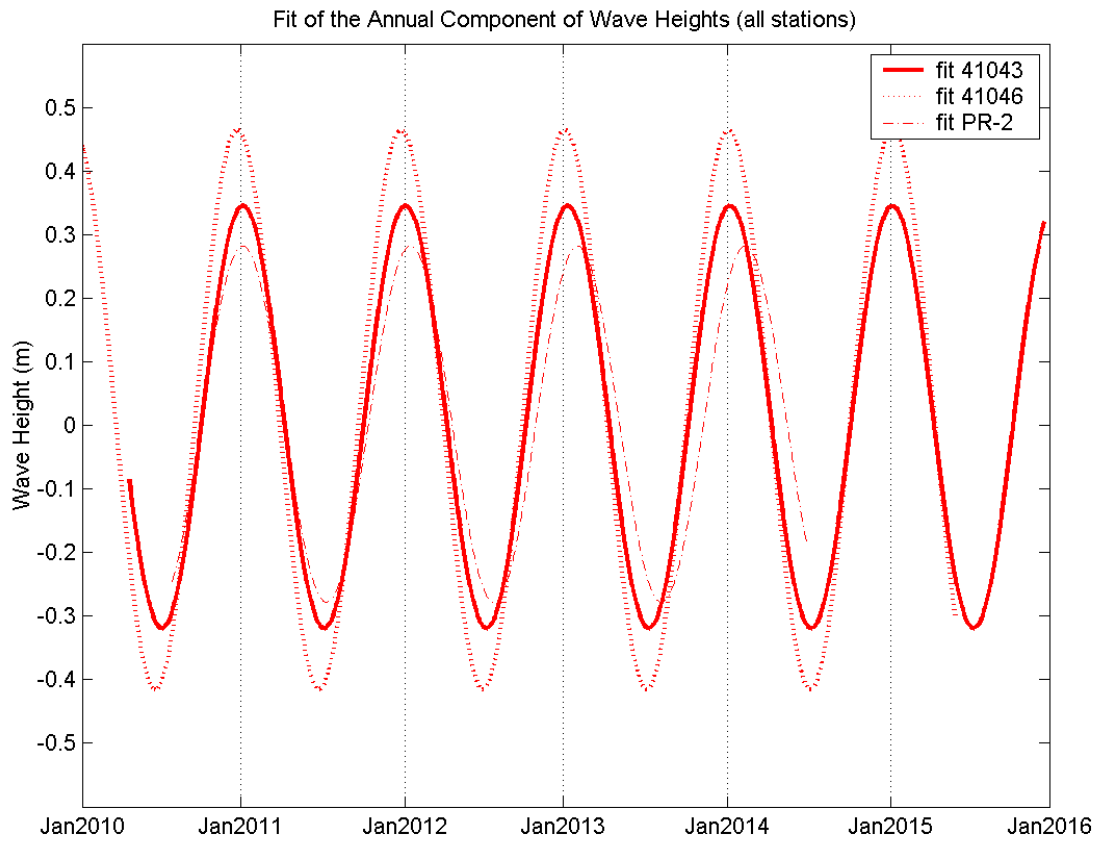
Goodness of fit:

SSE: 411.5

R-square: 0.7556

Adjusted R-square: 0.7556

RMSE: 0.1126



Dates of Maximum Height of the Fit		
NDBC 41046	NDBC 41043	CariCOOS PR2
19-Dec-10	2-Jan-11	1-Jan-11
25-Dec-11	4-Jan-12	15-Jan-12
29-Dec-12	6-Jan-13	28-Jan-13
4-Jan-14	8-Jan-14	12-Feb-14
10-Jan-15	11-Jan-15	

Figure 7.

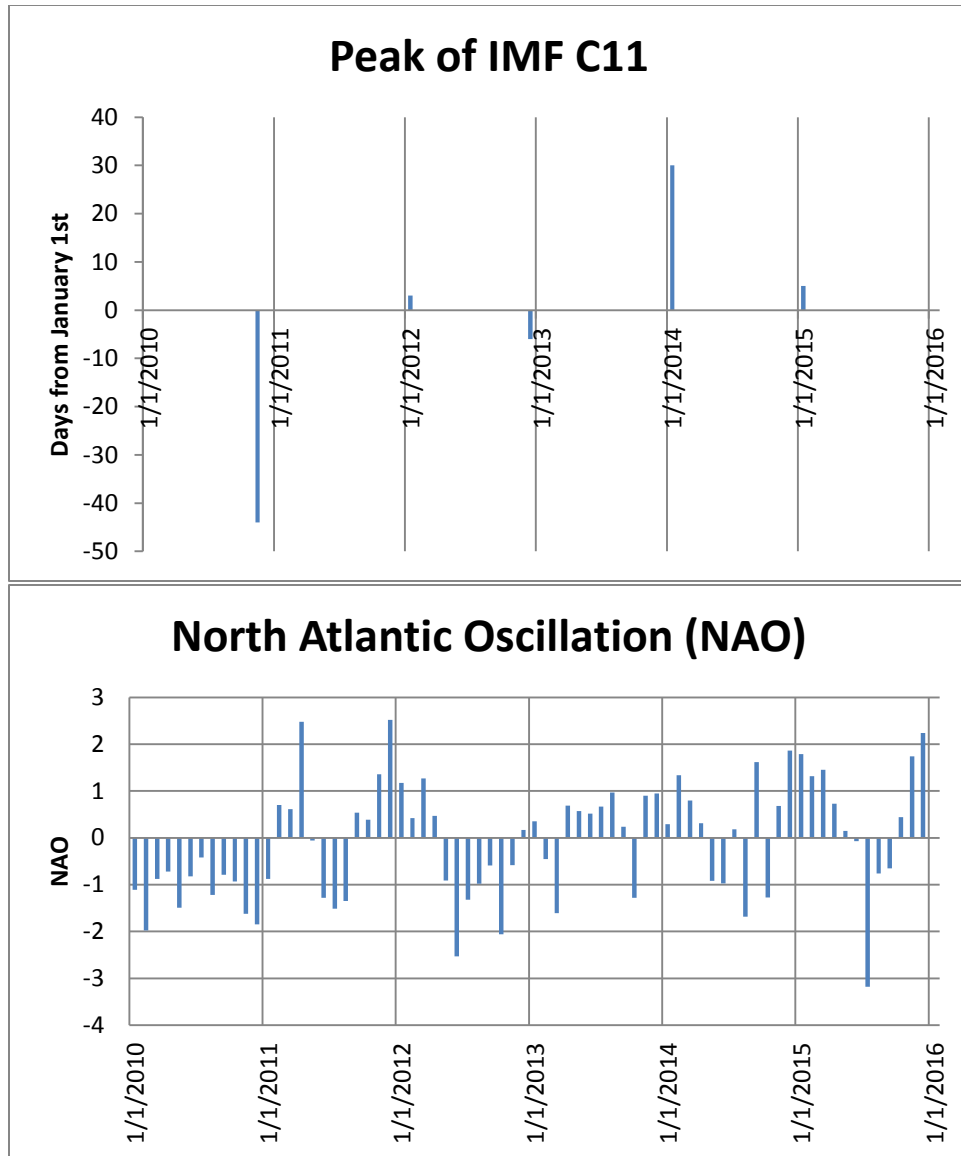
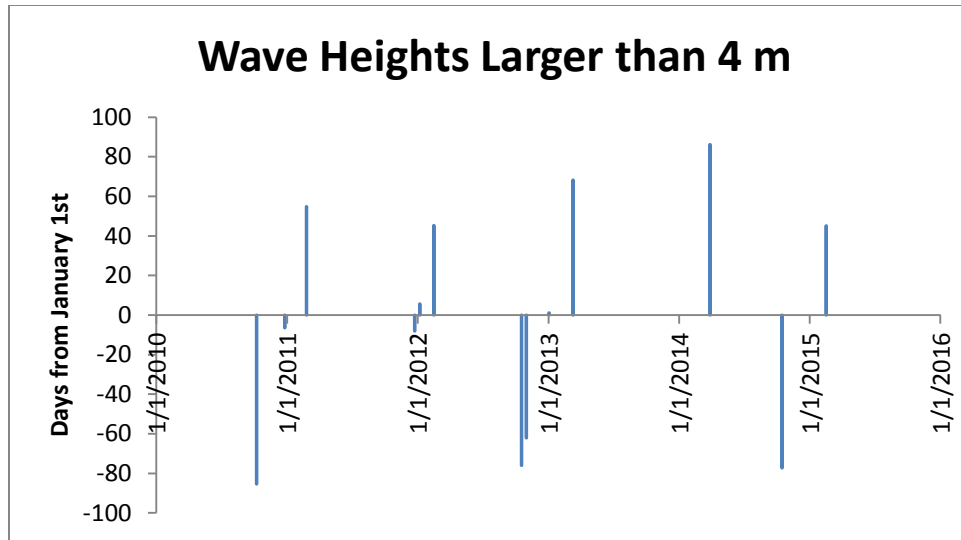


Figure 8A-8B



Events with Wave Heights Larger than 4 meters		
Wave Season	Date of the Event	Height (m)
2010-2011	10/7/2010 15:20	4.14
	12/25/2010 14:50	5.34
	2/24/2011 19:51	4.75
2011-2012	12/23/2011 22:09	4.27
	1/6/2012 17:51	4.27
	2/15/2012 5:36	4.45
2012-2013	10/17/2012 1:40	5.79
	10/30/2012 21:22	5.08
	1/2/2013 1:46	4.22
	3/10/2013 4:57	4.08
2013-2014	3/28/2014 6:08	4.55
2014-2015	10/15/2014 15:39	5.85
	2/15/2015 1:41	3.98

Figure 9.

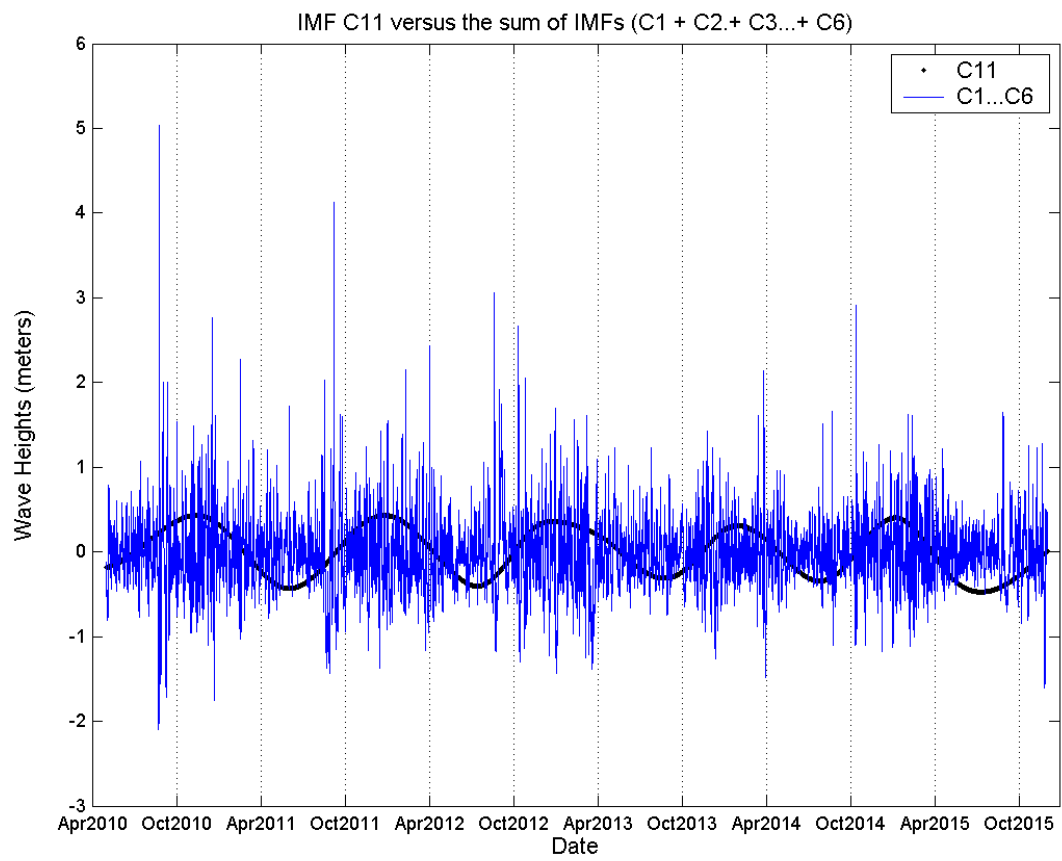


Figure 10.