The Hilbert-Huang Empirical Orthogonal Function and its application to the nonlinear-nonstationary internal tide

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Abstract

Hilbert-Huang Transform (HHT) and conventional empirical orthogonal function (EOF) analysis are combined to form HEOF analysis. HEOF analysis is based on modal decompositions of a covariance matrix calculated from the Empirical Mode Decomposition (EMD) of the original series. We present some basic formulations of HEOF and two implementation approaches: the first one consists in EMD, EOF, Hilbert Spectrum Analysis (HSA) and the second one in EMD, Hilbert Transform, and EOF, in that respective order. The first allows us to see the Hilbert amplitude spectrum for selected empirical modes and the second shows the amplitude of the intrinsic mode function's envelope for each empirical mode. Both approaches are used for the analysis of ADCP data collected at the shelf edge east of New Jersey. We estimated the response of the nonlinear nonstationary baroclinic tide to the spring-neap cycle of the barotropic tide. The results of HEOF analysis using both approaches reveal fluctuations in the baroclinic tidal amplitude cycle, with variable time scales between 10-20 days, and with an approximate time lag of 1-3 days respect the barotropic spring-neap cycle. In addition, a 5-10 day modulation cycle was observed. This amplitude cycle explains why strong internal tides forced by spring tides are followed by equally strong events a week later, during neap tides. HEOF analysis has proven useful in the analysis of internal tides and its use could be extended to other nonlinear geophysical time series associated with spatially coherent modal structures.

1. Introduction

Recently, a combination of wavelet and empirical orthogonal function (WEOF) analysis was applied in a study of internal tides (Wang et al. 2000). Their technique is particularly useful for the analysis of nonstationary geophysical signals. An alternative to wavelet analysis is the Hilbert Huang Transform (HHT) (Huang et al. 1998). It is capable of analyzing nonlinear nonstationary signals. In this paper, the Hilbert Huang Transform (HHT) and empirical orthogonal functions (EOF) are combined in HEOF analysis. This new technique is used to explore the internal tides.

HHT analysis consists of two parts: Empirical Mode Decomposition (EMD) and Hilbert Spectrum Analysis (HSA) (Huang et al. 1998). EMD decomposes any signal into a finite and small number of intrinsic mode functions (IMFs). In HSA the Hilbert transform is applied to each IMF yielding the time dependent instantaneous frequencies and amplitudes. Plotting the amplitudes along time-frequency axes traces the Hilbert Spectrum. HEOF consists of EMD analysis on any arrayed time series (such as ADCP measurements), followed by EOF decomposition and HSA. The conceptual innovation in HEOF is that we inserted EOF analysis between EMD and HSA. HEOF made possible to obtain separate Hilbert spectrums for each EOF mode. The technique is capable of detecting nonlinear internal waves possessing spatial coherent modal structures. But, as any EOF analysis, it does not guarantee that the statistical modes have a physical counterpart.

Internal tides are generated intermittently by the nonlinear interaction of the barotropic tide with the shelf break or slope topography. The dependence of baroclinic energy on the barotropic spring-neap cycle of the semidiurnal tide is of great interest. A number of investigators report the occurrence of stronger nonlinear internal activity during neap tides, when the barotropic currents are weaker (Pineda, 1995; Inall et al., 2000; Inall et al., 2001; Alfonso 2002). The 7-day lag is a striking and counterintuitive finding and has been reported in different locations around the world. Some authors have attributed this to an unclear relationship between nonlinear internal waves (NIWs) and the surface tide (Inall et al., 2001). Others proposed distant sources of internal waves and internal tides to explain it (Giese et al. 1982; Giese and Hollander, 1987; Wang et al., 2000).

The main objective of this paper is to estimate the response of the internal tide to the spring-neap cycle of the barotropic tide. The focus here is in the analysis of ADCP in the Mid-Atlantic Bight using HEOF.

2. Observations

In fall 2002 an extensive set of oceanographic measurements were made by investigators at the Naval Research Laboratory (NRL) in a region of the Mid-Atlantic Bight near 39.3 N, 72.7 W (Figure 1). The observations were made in support of NRL's Shallow Water Acoustics Experiment. Positions of moored ADCP's deployed from 30 September to 7 November, 2000 appear in Figure 1. CTD profiles were observed at numerous locations in the area and several moored thermistor chains were deployed during the early part of the ADCP record. One of the moored chains was located within about 150 m of ADCP-3 and hence provides complementary water column information.

The ADCP's were 307.2-kHz broadband "Workhorse Sentinels" manufactured by RD Instruments. The vertical bin size was 2 m for ADCP's 1 and 2, and 1 m for ADCP 3; valid currents were acquired from about 10 m beneath the surface to about 4 m above the bottom. Closer to the surface velocity data were contaminated by side-lobe interference. Near the bottom, a blanking distance of about 2.7 m from the transducer adds to the ~1 m height of the instrument. Water depths at the moorings ranged from 75 to 120 m. The sampling scheme used was a 1-min burst of 120 samples, which was repeated at 4-min intervals. The sampling interval was hence 4 min. Burst sampling was used to minimize aliasing of the measurement by surface gravity waves. Velocity data from each sample were resolved into northward and eastward components, then averaged over the burst. Velocities were rotated to correct for local magnetic deviation. The sample velocity uncertainty is estimated to be 1.5 cm/s or less.

In this paper we focus on analyzing the horizontal current velocities from ADCP3. Data from this location allow us to see the response of the internal tide in the shelf waters (~ 80 meters).

3. Analysis

Resampling

The eastward (U) and northward (V) currents were low-pass filtered (to avoid aliasing) and resampled at a one hour interval. High frequency internal waves were thus eliminated. This time interval is adequate for analysis of the internal tide.

Removal of the barotropic tides: harmonic analysis

Harmonic Analysis (HA) was applied to the velocity series individually for each depth. We applied a least squares method (Emery and Thomson 2001) to fit 16 constituents: M_2, K_1, O_1, S_2, MSf, N_2, Q_1, Mm including the shallow water constituents (MK_3, SK_3, M_4, MS_4, S_4, $2MK_5$, $2SK_5$, M_6), which are resolvable according to the Rayleigh criterion. The M_2 constituent dominates the barotropic signal; for example, at 30 m the M_2 amplitude is 10.4 cm s^{-1} followed by Mm, MSf with 3.8 cm s^{-1} and 3.6 cm s^{-1}, respectively. The dominant shallow water constituent is MS_4 with 0.5 cm s^{-1}. The mooring velocity time series was detided by subtracting the least squares fitted signal at each depth level. Hence, in the diurnal and semidiurnal band, this residual consists primarily of baroclinic tidal currents. The least squares fitted signals were vertically averaged to define the barotropic spring-neap cycle at the station.

Removal of near-inertial oscillations: Bandpass Filter

Energy at and near the local inertial frequency (f=1.27 CPD) was removed from the detided time series using a second order Butterworth Infinite Impulse Response (IIR) bandpass filter. The Butterworth filter guaranteed zero phase shifts and was designed carefully to keep short the span of the impulse response. The filter passband was chosen to isolate those frequencies between 1.1 and 1.4 CPD. This signal was subtracted from the detided signal at each depth level. What remains is a signal that consists mostly of baroclinic tidal and subtidal fluctuations.

Removal of terdiurnal and fourth diurnal oscillations: Low Pass Filter

Energy at frequencies higher than 2.6 CPD was removed from the detided inertial-removed currents. A carefully designed symmetric Butterworth filter of order three guaranteed zero phase distortion and minimized edge effects on the time series. The remaining signal, dominated by the semidiurnal baroclinic tide, was used in the HHT analysis.

Hilbert-Huang Transform (HHT) Analysis

In this paper we provide only a brief explanation of HHT analysis. It consists of a combination of Empirical Mode Decomposition (EMD) and Hilbert Spectrum Analysis (HSA). A detailed explanation can be found in Huang et al. (1998, 1999). The application of HHT for analyzing nonstationary and nonlinear geophysical series has been reported in a number of papers (Huang et al. 2001, Huang et al. 2000, Zhu et al. 1997). HHT performs time-frequency analysis of nonstationary signals, as do other techniques such as wavelets (Huang et al. 1998). However, HHT is better suited than wavelets for analyzing signals resulting from nonlinear processes. This advantage motivated us to choose HHT to analyze our data. EMD can separate the signal into orthogonal components with different time scales.

HHT analysis starts with empirical mode decomposition (EMD). This technique decomposes time series data into a finite number of intrinsic mode functions (IMFs) with time variable amplitudes and frequencies. The decomposition is orthogonal and adaptive[1]. Any function is an IMF if (1) in the whole data set, the number of extrema and the number of zero-crossings is either equal of differ at most by one, and (2) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima are zero. The second criterion means that the function has symmetric envelopes defined by local maxima and minima respectively. The process to achieve this decomposition is called “extrema sifting”. We limit the sifting to 100 times with a stoppage criterion of 5 times (CE(100,5), in the notation used by Huang, et al., 1998). This method allows us to (e.g.) easily separate the internal tide from the subtidal components (Figure 2). We focus on the first IMF, F_1, which represents the semidiurnal

[1] By adaptive we mean that the EMD decomposition adapts to the local variations of the data. Adaptive basis is indispensable for nonstationary and nonlinear data analysis.

baroclinic currents. An IMF is a band-limited signal which is amenable to the Hilbert transform. Hence, each IMF component can be expressed as an analytic signal:

$$c(t) = a(t)\exp(i\theta(t)), \qquad (1)$$

where $a(t)$ is the time varying amplitude and $\theta(t)$ is the time varying phase. These can be obtained from

$$a(t) = [X^2(t) + Y^2(t)]^{1/2}, \qquad \theta(t) = \arctan\left(\frac{Y(t)}{X(t)}\right) \qquad (2)$$

X(t) and Y(t) represent the real and imaginary part of c(t), respectively. The imaginary part was defined by the Hilbert transform of the original signal, X(t).

$$Y(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{X(t')}{t - t'} dt', \qquad (3)$$

where P indicates the Cauchy principal value. Y(t) is a version of the original real sequence with a 90° phase shift.

Variable $a(t)$ is the time varying amplitude of the analytic signal *c(t)*. It traces the envelope amplitude of the IMF component (Figure 3). $a^2(t)$ is then the relative energy of the modulation envelope.

The instantaneous frequency is defined as the time derivative of the instantaneous phase angle $\theta(t)$

$$\omega = \frac{d\theta}{dt}. \qquad (4)$$

Once we have decomposed the original series *X(t)* into *L* IMFs and applied the Hilbert transform to each, we can represent the original series as

$$X(t) = \sum_{\lambda=1}^{L} c_\lambda(t) = \sum_{\lambda=1}^{L} a_\lambda(t) \exp(i \int \omega_\lambda(t)\, dt). \qquad (5)$$

Using (1) and (4), we define the Hilbert amplitude spectrum as, $H_\lambda(\omega_\lambda(t), t) = a_\lambda(t)$ at $\omega_\lambda(t)$, for all modes λ. *H* is displayed as a contour plot in the time-frequency plane in Figure 8A. Alternatively, using the square of *a(t)* results in the Hilbert energy spectrum.

The foregoing is a synopsis of the HHT method. Details can be found in the cited references. In the remainder of this paper we discuss combined HHT and EOF analysis, as applied to the ADCP records.

Hilbert-Huang Empirical Orthogonal Function (HEOF) Analysis

First Approach

ADCP data consists of velocity time series at discrete depth levels. We represent the data as a matrix **X** with K rows and M columns (matrices are denoted with a bold font),

$$\mathrm{X} = X(t_k, z_m) = \begin{bmatrix} X(t_1,z_1) & X(t_1,z_2) & X(t_1,z_3) & \Lambda & X(t_1,z_M) \\ X(t_2,z_1) & X(t_2,z_2) & X(t_2,z_3) & \Lambda & X(t_2,z_M) \\ X(t_3,z_1) & X(t_3,z_2) & X(t_3,z_3) & \Lambda & X(t_3,z_M) \\ \mathrm{M} & \mathrm{M} & \mathrm{M} & & \mathrm{M} \\ X(t_K,z_1) & X(t_K,z_2) & X(t_K,z_3) & \Lambda & X(t_K,z_M) \end{bmatrix} \tag{6}$$

Let $X(t_k, z_m)$ denote the value of any variable at depth z_m (i=1, 2 ,3…M) and time t_k (k=1, 2, 3…K). Each column is a scalar time series of K observations for a given depth. The rows are vertical profiles; they represent the observations at M locations for time t_k (k=1, 2, 3…K). In our particular case each column is a time series of baroclinic speeds u (E-W speed component) for each depth level, z_m (i=1, 2, 3…M). Applying EMD decomposition on the first column of matrix **X**, generates L IMFs: $F_1(t_k,z_1)$, $F_2(t_k,z_1)$, $F_3(t_k,z_1)$… $F_L(t_k,z_1)$ (k=1, 2, 3…K) for the first depth, z_1. Repeating this for the rest of the columns (z_1, z_2, z_3 … z_M) results in a 3D matrix $\mathbf{F} \equiv \mathbf{F}_\lambda(t_k, z_m)$ (i=1, 2, 3… M) (k=1, 2, 3…K) (λ=1, 2, 3…L), where L is the total number of IMFs at each depth z_m (Figure 4).

We decompose the new 3D matrix **F** into a series of 2D matrices, slicing it at a particular component λ (Figure 5). $\mathbf{F}_\lambda$ has the dimensions K x M and its structure is similar to **X**. $\mathbf{F}_\lambda$ for λ=1 corresponds to the semidiurnal time scale in our data.

$$\mathrm{F}_l = \mathrm{F}_l(\mathrm{t_k}, \mathrm{z_m}) = \begin{bmatrix} \mathrm{F}_l(\mathrm{t_1},\mathrm{z_1}) & \mathrm{F}_l(\mathrm{t_1},\mathrm{z_2}) & \mathrm{F}_l(\mathrm{t_1},\mathrm{z_3}) & \Lambda & \mathrm{F}_l(\mathrm{t_1},\mathrm{z_M}) \\ \mathrm{F}_l(\mathrm{t_2},\mathrm{z_1}) & \mathrm{F}_l(\mathrm{t_2},\mathrm{z_2}) & \mathrm{F}_l(\mathrm{t_2},\mathrm{z_3}) & \Lambda & \mathrm{F}_l(\mathrm{t_2},\mathrm{z_M}) \\ \mathrm{F}_l(\mathrm{t_3},\mathrm{z_1}) & \mathrm{F}_l(\mathrm{t_2},\mathrm{z_2}) & \mathrm{F}_l(\mathrm{t_3},\mathrm{z_3}) & \Lambda & \mathrm{F}_l(\mathrm{t_3},\mathrm{z_M}) \\ \mathrm{M} & \mathrm{M} & \mathrm{M} & & \mathrm{M} \\ \mathrm{F}_l(\mathrm{t_K},\mathrm{z_1}) & \mathrm{F}_l(\mathrm{t_K},\mathrm{z_2}) & \mathrm{F}_l(\mathrm{t_K},\mathrm{z_3}) & \Lambda & \mathrm{F}_l(\mathrm{t_K},\mathrm{z_M}) \end{bmatrix} \tag{7}$$

The covariance matrix of $\boldsymbol{F_l}$ (between depths z_i and z_j) is

$$R_l(z_i, z_j) = \frac{1}{K}\sum_{k=1}^{K} F_l(t_k, z_i) F_l(t_k, z_j)$$
$$\text{where } i, j = 1, 2, 3 \ldots M \quad \text{depths.} \tag{8}$$

Following Kundu et al. (1975), we find the eigenvectors and eigenvalues of the covariance matrix $R_l(z_i,z_j)$,

$$\sum_{i=1}^{M} R_l(z_i, z_j)\varphi_n(z_i) = \lambda_n \varphi_n(z_j), \quad n = 1, 2, 3 \ldots M \tag{9}$$

The eigenvectors $\phi_n(z_i)$ are orthonormal,

$$\sum_{j=1}^{M} \phi_n(z_j)\phi_m(z_j) = \delta_{nm} \tag{10}$$

Here, $F_l\ (t_k, z_i)$ may be expanded by $\phi_n(z_i)$, such that

$$F_l(t_k, z_i) = \sum_{n=1}^{M} E_n(t_k)\varphi_n(z_i) \tag{11}$$

$$E_n(t_k) = \sum_{i=1}^{M} F_l(t_k, z_i)\varphi_n(z_i), \tag{12}$$

where the $E_n(t_k)$ are orthogonal and satisfy,

$$\sum_{j=1}^{M} E_n(t_k) E_m(t_k) = \lambda_m \delta_{nm}, \tag{13}$$

and λ_m is the total variance of mode *m*.

The $\phi_n(z_i)$, are the EOFs and the $E_n(t_k)$ are the principal components (PCs) of the expansion (11).

Following the above procedure (8-13) we obtain an EOF decomposition for each IMF, λ, resulting in a 3-D matrix (KxMxL) for each empirical mode; i.e. F^n, n=1,2,3…,M (see Figure 6). For IMF mode λ, the first 6 empirical modes explain 93.5% of the variance; Table 1 shows the distribution of variances in these modes.

The first IMF mode F_1^n (the semidiurnal band) EOFs and PCs are compared to dynamical modes (Kundu et al., 1975) based on a nearby buoyancy frequency profile. Correlations were high (>0.99 for mode 1 and >0.82 for mode 2) indicating that EOF modes are consistent with dynamical modes for the F_1 series for at least the first two empirical modes.

Applying equations 1-5 to matrix $\mathbf{F}^n$ yields the Hilbert spectrum for each depth and particular EOF mode, $\mathbf{H}^n \equiv H^n_\lambda(\omega_\lambda(t_k), t_k, z_i)$ (i=1, 2, 3… M) (k=1, 2, 3…K) (λ=1, 2, 3…L). This procedure, in contrast with the direct application of HSA to matrix **F,** reveals the specific time-frequency-depth patterns in each mode. This decomposition process is henceforth referred to as HEOF expansion.

Equation 7.3 from Huang et al. (1998), using our total data length, T = 39.6 d, and sampling rate, Δt = 0.0417 d, limits the maximum number of frequency cells to 188. The lowest frequency we can extract from the data is 1/T = 0.0253 CPD.

Examination of temperature data from the moored thermistor chain at the ADCP-3 location shows that the pycnocline depth oscillates between 30-40 meters. Hence, for a detailed look into the semidiurnal frequency band, we examine the Hilbert amplitude spectrum at 30 meters for the first three modes (n= 1, 2, 3); $\mathrm{H}^n_1(\omega_1(t_k), t_k, 30)$ is plotted in Figures 7B-D. In the first mode, frequencies are confined to a band limited by 0.6 and 3.0 CPD, but high amplitude events are in a narrower band between 1.5 and 2.3 CPD. Higher energy is evident on YD 281.9, 290.9 and YD 303.6. The second mode shows high amplitudes on YD 276.9, 284.6, 292.0, 303.4, 309.8, especially on YD 296.6. The third mode shows similar spread in frequencies, but less energy. Maximum energy occurs on YD 278.0.

Figure 8 shows a time lag of 1-2 days (i.e. age of the tide) between the time of spring barotropic currents and the time of maximum astronomical forcing during syzygy. The maximum range in barotropic currents occurred on YD 273.57, YD 288.59, YD 300.52. Based on these numbers, the observed age of the barotropic tide oscillated from -0.81 days up to 1.74 days (Table 2). The age of the tide determined from the age of phase inequality between the M_2 and S_2 phases obtained from HA of the currents is 0.81 days.

The semidiurnal IMF component matrix, $\mathbf{F_1}$ and the corresponding EOF modes: $\mathbf{F_1^1}$, $\mathbf{F_1^2}$, $\mathbf{F_1^3}$ are shown in Figure 9A-D. Intensification of the first EOF mode baroclinic currents occur around YD 274.0, 282.2, 291.0, 298.5, 303.6 and, YD 311.6 (Figure 9B). These events are separated from each other by 8.2, 8.8, 7.5, 5.1, 8 days, respectively. The separation times average 7.5 ± 1.4 days. The peak baroclinic currents occur between 0.4 to 3.0 days after the barotropic spring currents. But stronger baroclinic tides follow barotropic neap tides by 2.4-3.6 days. This is more obvious in the second EOF mode which shows strong currents between 30-50 m on YD 296, one day after neap currents. (Figure 9C). The time lag of the peak baroclinic currents relative to the barotropic spring currents suggests a 0.4-3 days travel from the generation area of the internal tide (Figure 10).

Second approach

In the first approach we examine the Hilbert Amplitude Spectrum for each empirical mode, empirical n-mode at each depth. An alternative approach is to construct a matrix, **A**, from amplitude, $a(t)$, of the analytical signal, c(t), derived from the Hilbert transform (Eq. 1-5) of each IMF component of the original data series X(t,z). Matrix **A** has dimensions (K x M x L), where L is the maximum number of IMF components.

$$A_\lambda = a_\lambda(t_k, z_m) = \begin{bmatrix} a_\lambda(t_1,z_1) & a_\lambda(t_1,z_2) & a_\lambda(t_1,z_3) & \Lambda & a_\lambda(t_1,z_M) \\ a_\lambda(t_2,z_1) & a_\lambda(t_2,z_2) & a_\lambda(t_2,z_3) & \Lambda & a_\lambda(t_2,z_M) \\ a_\lambda(t_3,z_1) & a_\lambda(t_3,z_2) & a_\lambda(t_3,z_3) & \Lambda & a_\lambda(t_3,z_M) \\ M & M & M & & M \\ a_\lambda(t_K,z_1) & a_\lambda(t_K,z_2) & a_\lambda(t_K,z_3) & \Lambda & a_\lambda(t_K,z_M) \end{bmatrix} \tag{14}$$

where $a_\lambda(t_k, z_m)$ (λ=1, 2, 3…L) (k=1, 2, 3…K) (m=1, 2, 3… M) is the amplitude of the analytical signal, c(t), for the chosen IMF component, λ.

As was done for matrix, **F** we slice the 3D matrix **A** along each IMF component, λ, and calculate EOFs and PCs.

In this case the covariance matrix of $\mathbf{A}_\lambda$ is

$$R_\lambda(z_i, z_j) = \frac{1}{K}\sum_{k=1}^{K} a_\lambda(t_k, z_i)\, a_\lambda(t_k, z_j)$$
$$\text{where } i, j = 1, 2, 3 \Lambda\ M \quad \text{depths} \tag{15}$$

Analogous to equation 9, 11 and 12 we obtain the eigenvectors and eigenvalues from the covariance matrix and $a_\lambda(t_k, z_i)$ can be expanded by $\phi_n(z_i)$,

$$a_\lambda(t_k, z_i) = \sum_{n=1}^{M} E_n(t_k)\,\phi_n(z_i) \tag{16}$$

where

$$E_n(t_k) = \sum_{i=1}^{M} a_\lambda(t_k, z_i)\,\phi_n(z_i) \tag{17}$$

Again, this procedure yields a set of 3-D matrices $\mathbf{A}^\mathbf{n}$ (n=1, 2, 3…M) for the empirical modes. For λ=1, the first 6 modes explain about 88.4% of the variance (Table 1). We plot the results for $\mathbf{A}_\mathbf{1}^\mathbf{n}$ (n=1, 2, 3) in Figure 11B-D. It shows the time-space distribution of the amplitude of the envelope that modulates the semidiurnal internal tide. The redder (bluer) areas indicate maximum (minimum) amplitude modulation.

Figure 11A shows matrix $\mathbf{A_1}$ (before EOF decomposition). Currents above 6 cm/s are prominent at 10-40 m including the pycnocline region (30-40 m). In addition strong currents are limited to the pycnocline (~30 m) on YD 296. After EOF analysis, the first mode, $\mathbf{A}_1^1$, is prominent on YD 273.8, 282.1, 291.0, 298.7, 303.6 and YD 311.6 (Figure 11B). These large amplitude events are separated from each other by 8.3, 8.9, 7.7, 4.9, 8.0 days, respectively. This result suggests that the baroclinic tidal amplitude has a mean timescale of 7.6 ± 1.6 days, half the time scale of the barotropic spring-neap cycle. This explains why large amplitude modulation (redder areas) on YD 282.1, 298.7 and 311.6 may follow the barotropic neap tides by a few days (Figure 12). Large amplitude modulation occurs 2.25, 3.83, and 2.74 days after the barotropic neap tides (Table 2). Large amplitudes are also seen in the second and third EOF modes (Figure 11C and 11D). These results demonstrate that large amplitude internal tides may occur after barotropic neap tides.

To provide an estimate of the fluctuation time scale of baroclinic tidal amplitude we further decompose each series in $\mathbf{A_1}$ into IMFs, to allow a quantitative estimate of modulation frequencies. Here, the sifting results in only five IMFs. The resulting Hilbert amplitude spectrum (Figure 13A) traces (for example) the modulation envelope of $F_1(t_k,30m)$. The resulting Hilbert spectrum shows a strong fortnightly component in the modulation (Figure 13A), with the fortnightly cycle contained mainly in the last IMF. By selecting and grouping the instantaneous frequencies with nonzero amplitudes (values > 0.05) we compute an empirical probability density estimate (PDE) of $\omega_1(t)$ for each depth. The kernel density estimation is based on optimal normal bandwidth (Bowman and Azzalini, 1997). Merging all of them into one plot helps define the time scale of the baroclinic tidal amplitude (Figure 13C). Most of the peaks are concentrated between 0.05 CPD and 0.1 CPD (20-10 days). The average of all the PDE's (traced by the thick line) shows a peak centered on 0.07 CPD (14 days). The strong fortnightly-scale may obscure the other time scales of modulation. If we remove the last IMF this problem can be overcome. The new Hilbert spectrum is shown in Figure 13B. The new PDE's shift to the right showing most peaks between 0.1 CPD and 0.2 CPD (Figure 14D). These peaks correspond to modulations of the semidiurnal internal tide ranging between 5-10 days. The average of all PDE's shows a modulation frequency of 0.135 CPD (7.4 days). These results show that the fluctuation of baroclinic tidal amplitude does not show a regular 2-week cycle but instead a variable one.

4. Summary and Conclusions

Baroclinic tides in shelf waters are studied using data collected from one ADCP in the Mid-Atlantic Bight. The response of the baroclinic currents to the spring-neap cycle of the barotropic currents is explored using two approaches of HEOF analysis. Both approaches reduce complexity of HHT analysis of the ADCP data.

HEOF analysis is based on modal decompositions of a covariance matrix calculated from the EMD analysis of the original series. We have shown two ways to implement it: the first approach consisted of EMD, EOF, HSA and the second approach of EMD, Hilbert transform, EOF, in that respective order. The first allows us to see the Hilbert amplitude spectrum for selected empirical modes and the second shows the amplitude of the intrinsic mode function's envelope for each empirical mode. One is a complement of the other so both approaches should be used to provide a complete analysis of the ADCP data.

This method allows the detection of spatially coherent modal structures of (e.g.) nonlinear internal waves. The second approach is useful to describe the temporal evolution of the envelope modulating the amplitude of the waves. In addition, EOF analysis identifies the spatial coherence of this modulation process.

Both approaches reveal a variable baroclinic tidal amplitude (between 10-20 days) with an approximate time lag of 0.4-3 days respect the barotropic spring-neap cycle probably due to the distance from the generation area. In addition, a 5-10 day modulation cycle is present. The second and third EOF modes show increased internal tide 0.4-3 days after barotropic neap tides. A baroclinic tidal amplitude cycle of half the barotropic spring-neap cycle has been reported on current measurements near the Canary Basin (Siedler and Paul, 1991). They found that the first dynamic mode shows time scales of the order of 5 days, similar to the time scales observed in $\mathbf{A_1}$. In addition, our findings support some observations of stronger mixing during neap tides rather than during spring barotropic tides (Rippeth and Inall, 2002; Alfonso, 2002). Also a 5-10 d baroclinic cycle could explain observations of large amplitude coastal seiche phenomena observed during neap tides without the necessity to invoke the hypothesis of far generated internal waves (Giese 1982). Finally, there are alternative explanations for the concurrence of strong baroclinic signals and weak barotropic tides. Using a numerical model, Gerkema (2002) established that a baroclinic spring-neap cycle may have large phase shifts with respect to the barotropic spring-neap cycle due to the different trajectories of the M_2 and S_2 internal beams and the local difference in phase between them at different locations. Gerkema findings can explain a phase shift in a 2-week baroclinic spring-neap cycle but it seems more difficult that it could explain a 5-10 day cycle.

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6. References

Alfonso, E., 2002. *Variabilidad temporal de la produccion primaria fitoplanctonica en la estacion CaTS (Caribbean Time-Series Station): Con enfasis en el impacto de la marea interna semidiurna sobre la produccion.* Ph. D. Dissertation. Department of Marine Sciences, University of Puerto Rico, Mayagüez, Puerto Rico. UMI publication AAT 3042382, 407 pp.

Bowman, A. W., Azzalini, A. 1997. Applied Smoothing Techniques for Data Analysis, Oxford University Press, 208 pp.

Emery, W. J., and R. E. Thompson. 2001. Data analysis methods in physical oceanography. 2nd ed., Elsevier, New York, 638 pp.

Gerkema, T. 2002. Application of an internal tide generation model to baroclinic spring-neap cycles. Journal of Geophysical Research 107(C9), 3124.

Giese, G. S., Hollander, R.B., 1987. The relationship between coastal seiches at Palawan Island and Tide-Generated Internal Waves in the Sulu Sea. Journal of Geophysical Research 92(C5), 5151-5156

Giese, G. S., Hollander, R.B., Francher, J.E., Giese, B.S., 1982. Evidence of coastal seiche excitation by tide-generated internal solitary waves, Geophysical Research Letters 9, 1305-1308.

Huang, N. E., Chern, C. C., Huang, K., Salvino, L. W., Long, S. R. and K. L. Fan. 2001. A new spectral representation of earthquake date: Hilbert spectral analysis of station TCU129, Chi-Chi, Taiwan, 21 September 1999.

Huang, N. E., Shih, H. H., Shen, Z., Long, S. R., and K. L. Fan. 2000. The ages of large amplitude coastal seiches on the Caribbean Coast of Puerto Rico. Journal of Physical Oceanography 30(8), 2001-2012.

Huang, N. E., Shen, Z., and S. R. Long, M. C. Wu, H. H. Shih, Q. Zheng, N-C. Yen, C. C. Tung, and H. H. Liu. 1998. The empirical mode decomposition and the Hilbert spectrum fon nonlinear and non-stationary time series analysis, Proc. R. Soc. London Series A 454: 903-995.

Huang, N. E., Shen Z. and S. R. Long. 1999. A new view of nonlinear water waves: the Hilbert Spectrum, Annual Review of Fluid Dynamics 31:417-457.

Inall, M. E., Rippeth, T. P. and T. J. Sherwin. 2000. Impact of nonlinear waves on the dissipation of internal tidal energy at a shelf break. Journal of Geophysical Research 105(C4): 8687-8705.

Inall, M. E., Shapiro, G. I. and T. J. Sherwin. 2001. Mass transport by non-linear internal waves on the Malin Shelf. Continental Shelf Research 21: 1449-1472.

Kundu, P. K., J. S. Allen and R. L. Smith. 1975. Modal decomposition of the velocity field near the Oregon coast, J. Phys. Oceanogr., 5, 683-704.

Pineda, J. 1995. An internal tidal bore regime at nearshore stations along western U.S.A.: Predictable upwelling within the lunar cycle. Continental Shelf Research 15(8): 1023-1041.

Rippeth, T. P. and M. E. Inall, 2002. Observations of the internal tide and associated mixing across the Malin Shelf, Journal of Geophysical Research 107(C4), 3028.

Siedler, G., U. Paul. 1991. Barotropic and baroclinic tidal currents in the eastern basins of the North Atlantic. Journal of Geophysical Research 96(C12):22259-22271.

Wang J., Chern, C. C., and A. K. Liu. 2000. The wavelet empirical orthogonal function and its application to the analysis of internal tides. Journal of Atmospheric and Oceanic Technology 17: 1403-1420.

Zhu, X., Shen Z., Eckermann, S. D., Bittner, M., Hirota I., and J-H. Yee. 1997. Gravity wave characteristics in the middle atmoshphere derived from the Empirical Mode Decomposition method. Journal of Geophysical Research 102(D14):16545-16561.

Figures Captions

Figure 1. ADCP locations are indicated by triangles. Contours are in meters A moored thermistor chain was deployed at the ADCP-3 location for about 12 days.

Figure 2. IMF sifting of the one hour re-sampled baroclinic current, u(t), at 30 meters depth. F_1 shows a semidiurnal time scale and may represent the SD nonlinear baroclinic tide. Largest currents were observed on YD 296.

Figure 3. A closer look at the IMF component F_1 (for u(t)) corresponding to the semidiurnal internal tide (continuous line). The analytic function, c(t), that results from the Hilbert transform traces the envelope of the SD baroclinic tide.

Figure 4. Diagram explains the generation of matrix **F** from the original data in matrix **X**. First, perform EMD analysis on the time series at each depth. This process generates L IMF's, for each of the M depth elevels. Second, group the results in the matrix **F.**

Figure 5. Decomposition of the 3D matrix **F** into a series of 2D matrices $\mathbf{F}_\lambda$ slicing it at a each IMF component, λ, where λ=1, 2, 3…L. The dimension of $\mathbf{F}_\lambda$ is K x M.

Figure 6. EOF decomposition of the 2D matrices: $\mathbf{F_1}$**,** $\mathbf{F_2}$**,** $\mathbf{F_3}$**…** $\mathbf{F_L}$.

Figure 7(A) Hilbert spectrum, $H(\omega_\lambda(t_k), t_k, 30)$**,** before HEOF decomposition. Yellow-red patches inside the semidiurnal band (1.5-2.5 CPD) occurred on YD 286, 297 and 304 and indicates intensification of the SD baroclinic tide. **(B)** Hilbert spectrum, $H_1^1(\omega_\lambda(t_k), t_k, 30)$ after HEOF decomposition using the first approach. **(C)** Same for mode two: $H_1^2(\omega_\lambda(t_k), t_k, 30)$. **(D)** Same for mode three: $H_1^3(\omega_\lambda(t_k), t_k, 30)$.

Figure 8. Barotropic spring-neap cycle versus the synodic cycle. The time interval between the lunar phase and the corresponding maximum (or minimum) range in tidal currents defines the age of the tide.

Figure 9.(A) Plot of $\mathbf{F_1}$, before HEOF decomposition. **(B)** Plot of $\mathbf{F_1^1}$ after HEOF decomposition using the first approach. **(C)** Same for mode two: $\mathbf{F_1^2}$**.** **(D)** Same for mode three: $\mathbf{F_1^3}$**.**

Figure 10.(A) Plot of $\mathbf{F_1^1}$ shows the semidiurnal baroclinic currents (u). **(B)** Barotropic spring-neap cycle.

Figure 11. (A) Plot of $\mathbf{A_1}$, before HEOF decomposition. **(B)** Plot of $\mathbf{A_1^1}$ after HEOF decomposition using the second approach. **(C)** Same for mode two: $\mathbf{A_1^2}$**.** **(D)** Same for mode three: $\mathbf{A_1^3}$**.**

Figure 12. (A) Plot of $\mathbf{A_1^1}$ shows the first EOF mode of the envelope $\mathbf{A_1}$**.** **(B)** Barotropic spring-neap cycle.

Figure 13. (A) Hilbert spectrum after applying EMD decomposition on the envelope, $\mathbf{A_1}$**.** **(B)** Same as figure 14A, but the IMF with fortnightly time scale was removed. **(C)** **Empirical** probability density functions (PDE's) of the envelope instantaneous frequencies for each depth. **(D) Empirical** probability density functions (PDE's) of the envelope instantaneous frequencies once the fortnightly-scale IMF was removed.