Wind-driven Western Boundary Ocean Currents in Terran and Superterran Exoplanets

By

Edwin Alfonso-Sosa, Ph.D.

Ocean Physics Education, Inc.

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Introduction

Simple models of oceanic general circulation attribute the motion entirely to wind action, in particular wind stress on the sea surface. These models are homogeneous in density and therefore completely ignore the dynamical effects of ocean's stratification, and most often the models assume a flat bottom ocean basin and a simple shape of the basin's perimeter. Nevertheless the models are remarkably successful in describing the general nature of the ocean's wind-driven horizontal circulation, in particular the intensification of the western boundary currents (Munk, 1950; Stommel, 1948) (Figure 1). This westward intensification is characterized for having the fastest surface ocean currents in the planet. Each of the Earth's oceans manifests the westward intensification (Ex: Gulf Stream C., Kuroshio C., Brazil C., Agulhas C.), although the basins differ in the details of shape, bottom topography and the pattern of the wind stress. This fact suggests that the general ocean circulation moves according to powerful, yet simple constraints. Homogenous models are successful because they can grasp much of the essential physics of the general oceanic circulation. The same basics physics must apply in Terran and Superterran exoplanets and explain their oceanic circulation. The simplicity of the homogenous models makes them fully adaptable to other oceans in exoplanets. This simplicity facilitates the easy comparison of results, after inputting the measureable properties of exoplanets such as: mean radius and mean angular velocity. These two are independent variables of the planet's vorticity gradient. In other words, the variation of the Coriolis parameter with latitude controls the width of the western boundary layer and the magnitude of the western boundary current. Our model is **not** intended to represent a real ocean in these exoplanets but to investigate the relationship between ocean boundary currents and exoplanet's size and rotation. We derived some linear mathematical relationships between the two, which can be useful to characterize the western boundary currents in these exoplanets. Now is possible to collect planet size and rotation data from online catalogs such as the Habitable Exoplanets Catalog (HEC), Extrasolar Planet Encyclopedia and NASA Exoplanet Archive.

Methodology

Consider the 1-D model of wind driven flow over a homogenous ocean contained in a flat-bottomed rectangular basin, with only lateral friction as a retarding force (no bottom friction). The governing equation is:

$$\frac{\partial \zeta}{\partial t} + \beta \frac{\partial \psi}{\partial x} = -\frac{\tau_0 \pi}{\rho D L_y} + A\left(\frac{\partial^2 \zeta}{\partial x^2} - \frac{\pi^2}{L_y^2}\zeta\right)$$

$$\frac{\partial^2 \psi}{\partial x^2} - \frac{\pi^2}{L_y^2}\psi = \zeta$$

The boundary conditions were no-slip:

$$\frac{\partial \psi}{\partial x} = 0 \quad v(x=0) = v(x=L_x) = 0 \quad and \quad \psi(x=0) = \psi(x=L_x) = 0$$

The stream function solution, ψ, and the current speed, v, were calculated for the following fixed parameters:

$$L_x = 2000km \quad L_y = 1000km \quad D = 1000m \quad \Delta x = \Delta y = 20km \quad \Delta t = 30000s$$

$$\rho = 1000\frac{kg}{m^3} \quad \tau_0 = 0.1\frac{N}{m^2} \quad A_H = 5000m^2s^{-1}$$

The wind stress was ramped over in 20 steps. Changes in wind stress due to changes in wind velocity and ocean surface roughness were not included in the model. The time dependent problem was solved using Forward Time finite difference and a variable weight implicit scheme for the lateral viscosity term with θ = 0.5 (Crank-Nicholson). We started with no flow, ζ=ψ=0, and integrated to steady state using the total kinetic energy of the current,

$$KE = \frac{1}{2}\int_0^{L_x}\left(u^2 + v^2\right)dx$$

as the convergence criterion. The stop criterion was $\Delta KE \leq 0.001\ m^3\ s^{-2}$.

The only input variable into this homogeneous model was the value of the planet's vorticity gradient, Beta (Pedlosky, 1987),

$$\beta = \frac{\partial f}{\partial y} = \frac{2\Omega\cos\theta}{R}, \quad f = 2\Omega\sin\theta$$

The variable *f* is the Coriolis parameter, the component of the planetary vorticity normal to the planet's surface. Omega, Ω, is the planet's uniform angular velocity. The latitude, θ, was fixed to 30°. The above mathematical expression is valid only when the horizontal length, L, is smaller than the mean planet radius, L < O(R). In the case of planet Earth, the value of $R = R_0 = 6371$ km and $\Omega = \Omega_0 = 7.2921e\text{-}005$ rad s^{-1}. We used the zero subscript to denote planet Earth's values. From the above mathematical expression we obtained a series of different values of the planet's vorticity gradient, β, for each planet radius (Figure 2, Top). Each individual β-value was inputted into the model and run until it reached steady state.

Planet's Radius $\times R_0$	Beta ($m^{-1}\ s^{-1}$) β
0.9	2.20E-11
1.0	1.98E-11
1.3	1.53E-11
1.5	1.32E-11
1.8	1.10E-11
2.0	9.91E-12

In addition, we calculated various values of β, after changing the planet's angular velocity and keeping fixed the planet's radius ($R=1R_0$) (Figure 2, Bottom).

Planet's Angular Velocity $\times \Omega_0$	Beta ($m^{-1}\ s^{-1}$) β
0.7978	1.58E-11
0.9032	1.79E-11
1.0000	1.98E-11
1.1100	2.21E-11

The previous approach resulted in two sets of results: one for the case of changing the planet radius and a second one for changing the planet angular velocity. As expected, it shows an increase in the planet's vorticity gradient, β, as the planet's radius decreases and as the planet's angular velocity increases. We chose values that are a little below and above the Earth's values. Terran and Superterran exoplanets could show similar values.

For each value of β the model was run until it reached the steady state criterion: $\Delta KE \leq 0.001\ m^3\ s^{-2}$. Figure 3 shows the planet's radiuses and angular velocities (rotation speeds) versus the steady state total kinetic energy (KE) of the current, respectively. The steady state total KE of the current increases as the planet's radius increases. For the contrary, the total KE decreases as the planet's angular velocity increases.

Results

Figure 4 shows an example of the model's output: the normalized stream function ψ and the normalized meridional velocity, v, from x=0 to $x=L_x$=2E+06 m; for two different planet radiuses: $1R_0$ and $2R_0$. The x-position at the maximum value of the meridional velocity, v_{MAX}, shifted east some 18.9 km when the radius was doubled. This position represents the eastern limit of the western boundary layer. Doubling the planet radius had the effect of widening the boundary layer by 23.5%. The boundary-layer width for each β value is shown in the table below. Also, the theoretical boundary-layer width, l_*, can be calculated by (Pedlosky J., 1987):

$$l_* = \left(\frac{A_H}{\beta}\right)^{\frac{1}{3}}$$

and the corresponding l_* values are included in the table below. In average, the numerical value is greater than the theoretical one by 13.5±5.3 km.

Planet Radius ($\times R_0$)	β ($m^{-1}\,s^{-1}$)	Iterations	v_{MAX} ($m\,s^{-1}$)	X @ $v=v_{MAX}$ (m)	X @ v=0 (m)	l_* (m)
0.9	2.20E-11	3239	0.21523	7.85E+04	2.13E+05	6.10E+04
1.0	1.98E-11	2339	0.23134	8.03E+04	2.22E+05	6.32E+04
1.3	1.53E-11	1536	0.27148	8.03E+04	2.43E+05	6.90E+04
1.5	1.32E-11	1517	0.29330	8.03E+04	2.52E+05	7.23E+04
1.8	1.10E-11	1452	0.32560	8.41E+04	2.69E+05	7.69E+04
2.0	9.91E-12	1273	0.34658	9.92E+04	2.81E+05	7.96E+04

The table above shows that as the planet's radius increases also does the peak value of the western boundary current, v_{MAX}. In addition, as the radius increases the boundary layer widens. There is an inverse relationship between the planet's vorticity gradient, β, and v_{MAX}.

Figure 5 shows the normalized stream function ψ and the normalized meridional velocity, v, for two different planet's angular velocities: $0.8\Omega_0$ and $1.1\Omega_0$. Increasing the angular velocity, Ω, has the effect of shifting 24.6 km west and 18.9 km west each curve, respectively. Increasing Ω decreases the western boundary layer width. For this particular case it was reduced by 2.4%. In average, the numerical value of the layer's width is greater than the theoretical one by 15.5±2.43 km. The table below shows that an increase in the planet's angular velocity caused a reduction in the peak value of the western boundary current. Again it shows an inverse relationship between the planet's vorticity gradient, β, and v_{MAX}. Increases in β caused the reduction of the planet's maximum value of the western boundary current.

Angular Velocity ($x\ \Omega_0$)	β ($m^{-1}\ s^{-1}$)	Iterations	v_{MAX} ($m\ s^{-1}$)	X @ $v=v_{MAX}$ (m)	X @ v=0 (m)	l_* (m)
0.7978	1.58E-11	1396	0.2652	8.03E+04	2.37E+05	6.81E+04
0.9032	1.79E-11	2024	0.2462	8.03E+04	2.28E+05	6.54E+04
1.0000	1.98E-11	2339	0.2313	8.03E+04	2.22E+05	6.32E+04
1.1100	2.21E-11	4057	0.2159	7.85E+04	2.13E+05	6.10E+04

If we normalize each planet's v_{MAX} value by the Earth's v_{MAX} value, and plot it against the planet's radius, we can find a useful linear relationship between the two. Figure 6 (top) shows the normalized meridional speed of the western boundary current for various planet radiuses. A linear relationship between the planet radiuses ($0.9R_0$-$2.0R_0$) and the maximum western boundary current was found:

$$\frac{v_{\max Planet}}{v_{\max Earth}} = 0.5\left(\frac{R}{R_0}\right) + 0.5$$

this simple mathematical relationship applies only inside the planet radius range between $0.9R_0$ and $2.0R_0$. This range corresponds to Terran and Superterran exoplanets. If the exoplanet has a radius equal to $2R_0$ then the $v_{MAX\ Planet}=1.5\ v_{MAX\ Earth}$, representing an increase of about 50%.

Figure 6 (bottom) shows the normalized meridional speed of the western boundary current for various angular velocities. A linear relationship was found that is valid inside the planet's angular velocity range of $0.8\Omega_0$ and $1.1\Omega_0$.

$$\frac{v_{\max Planet}}{v_{\max Earth}} = -0.68\left(\frac{\Omega}{\Omega_0}\right) + 1.68$$

Both equations are simple but powerful because allows to have a good estimate of the maximum western boundary currents based only on the planet's radius or angular velocity and these two parameters can be calculated by precise astronomical observations.

A similar linear relationship between the planet radius and the width of the western boundary layer, l_*, was found:

$$\frac{l_{*Planet}}{l_{*Earth}} = 0.27\left(\frac{R}{R_0}\right) + 0.73$$

If the exoplanet has a radius equal to $2R_0$ then the $l_{*\ Planet}=1.27\ l_{*\ Earth}$, representing a width increase of about 27%.

Discussion

It is obvious that a v_{MAX} value of 0.23 m s^{-1} for planet Earth (R_0, Ω_0) is far less than the maximum measured speeds (2 m s^{-1}) in western boundary currents such as the Gulf Stream. It is evident that in our model we are not interested in the actual *in-situ* real value but in a simple model that could represent the western boundary currents in a general way. This simplicity allowed us to easily adapt it to other planet sizes and rotation speeds. The model was limited to a specific range of planet sizes ($0.9R_0$-$2.0R_0$) and angular speeds ($0.8\Omega_0$-$1.1\Omega_0$) because outside of that range the steady state solutions were not achievable. Remember that we set very strict constraints such as a fixed A_H and value of dt. The only variable quantity was β, which was a function of the planet radius and angular velocity. All these restrictions made possible a fair comparison and to generate a linear relationships between these fundamental planet properties and the boundary currents properties. For example doubling the planet radius but keeping the same angular speed as in planet Earth results in an increase of 50% in the maximum current speed. If the maximum western boundary current speed in planet Earth is 2 m s^{-1} then the hypothetical planet will have a speed 1.5 times faster, 3 m s^{-1}. Also doubling the planet radius has the effect of widening the western boundary current by 27%. To the contrary, increases in angular velocity reduced the current speed and narrowed the boundary layer. Increasing the angular velocity from $0.8\Omega_0$ to $1.1\Omega_0$ reduced the boundary current speed by 18.6 % and narrowed it by 2.4%. But the theoretical boundary-layer width, l_*, was reduced by 10.5%. The above results demonstrate that the exoplanet's radius and angular velocity are key factors to determine the properties of the western boundary currents in that planet.

Long-term observations of the Gulf Stream support that strong western boundary currents are permanent features of ocean circulation. Two-decades of directly measured velocity across the Gulf Stream Current show evidence of the long-term stability of the Gulf Stream Transport (Rossby et al., 2014). Even in the current Global Warming scenario. This long-term stability is a reflection of the robust physics that control this type of ocean currents. Despite the atmospheric oscillations and changes in the wind field, the Gulf Stream Current keeps flowing without major changes in transport. The basic physics that sustain these boundary currents clearly suggests that they must be present in Terran and Superterran planets.

The planet's vorticity gradient, β, is responsible for the westward intensification and this result can be explained in terms of conservation of vorticity (Pond and Pickard, 1987). Wind stress puts vorticity (i.e. spin) into the ocean but lateral friction is required to take it out. Strong lateral friction in the west is necessary to take out the vorticity, and for this strong friction to occur strong currents with strong shear are needed. Once the value of β increased due to decreases in planet radius or due to increases in angular velocity, the westward intensification increased and the lateral friction reduced the vorticity in the circulation to reach a steady state. This explains why as β increases the lateral friction, it reduces the value of v_{MAX} and the boundary layer width. Another way to visualized this is to push our model into an extreme case: a tidally-locked Earth. For this particular case one earth day equals one sidereal year, $\Omega=0.0027\Omega_0$. This extremely low angular velocity results in a very low value of β and **no** westward intensification (i.e. well-spaced concentric streamlines) (Figure 7, Top). The lateral friction is so weak that the currents can increase up to a v_{MAX}=0.71 m s^{-1}, a 208% increase. It shows almost equal maximum speeds for northward and southward currents far away from the boundaries (Figure 7, Bottom). The steady state KE reached 4.4E+05 m^3 s^{-2}; it is 72 times the actual Earth's KE value.

Conclusion

The planetary vorticity gradient controls the maximum speed and width of the western boundary current. A larger than Earth's value of the vorticity gradient means a stronger westward intensification, narrowing the boundary layer width and decreasing the maximum current speed due to more lateral friction. A smaller than Earth's value means a weaker westward intensification, widening the boundary layer width and increasing the maximum current speeds due to less lateral friction. In other words, exoplanets with radiuses equal to Earth, but with planetary angular speeds faster than Earth have a stronger westward intensification than Earth and vice versa. Exoplanets with planetary angular speeds equal to Earth, but with smaller radiuses than Earth have a stronger westward intensification than Earth and vice versa.

For very slow planetary angular speeds the western intensification disappears and the fastest currents move far from the margins, toward the interior of the ocean basin. As a planet decreases its planetary vorticity gradient then the westward intensification in the basin decreases and the interior circulation increases.

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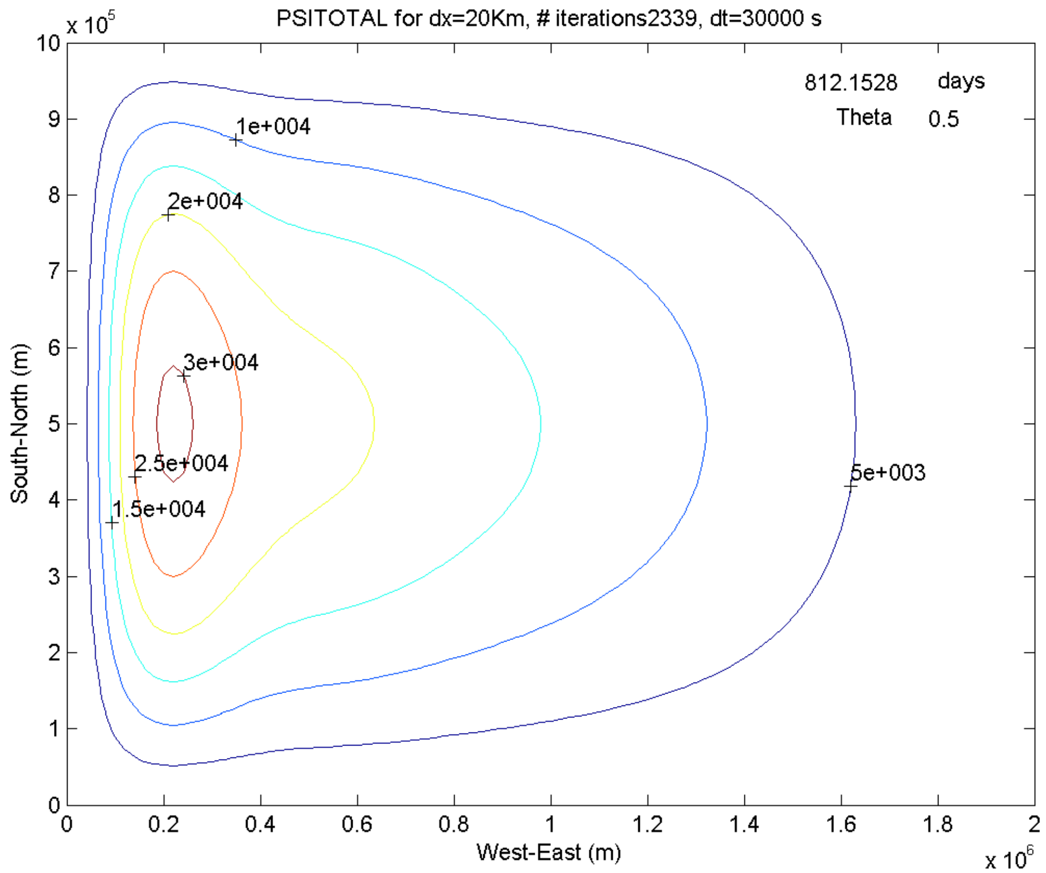


Figure 1. Flow pattern (streamlines) for simplified wind-driven circulation in a flat-bottom rectangular ocean with the Coriolis parameter increasing linearly with latitude. Note the western intensification, where the streamlines are close together in the west margin. Where the streamlines are close the flow is swift and vice versa.

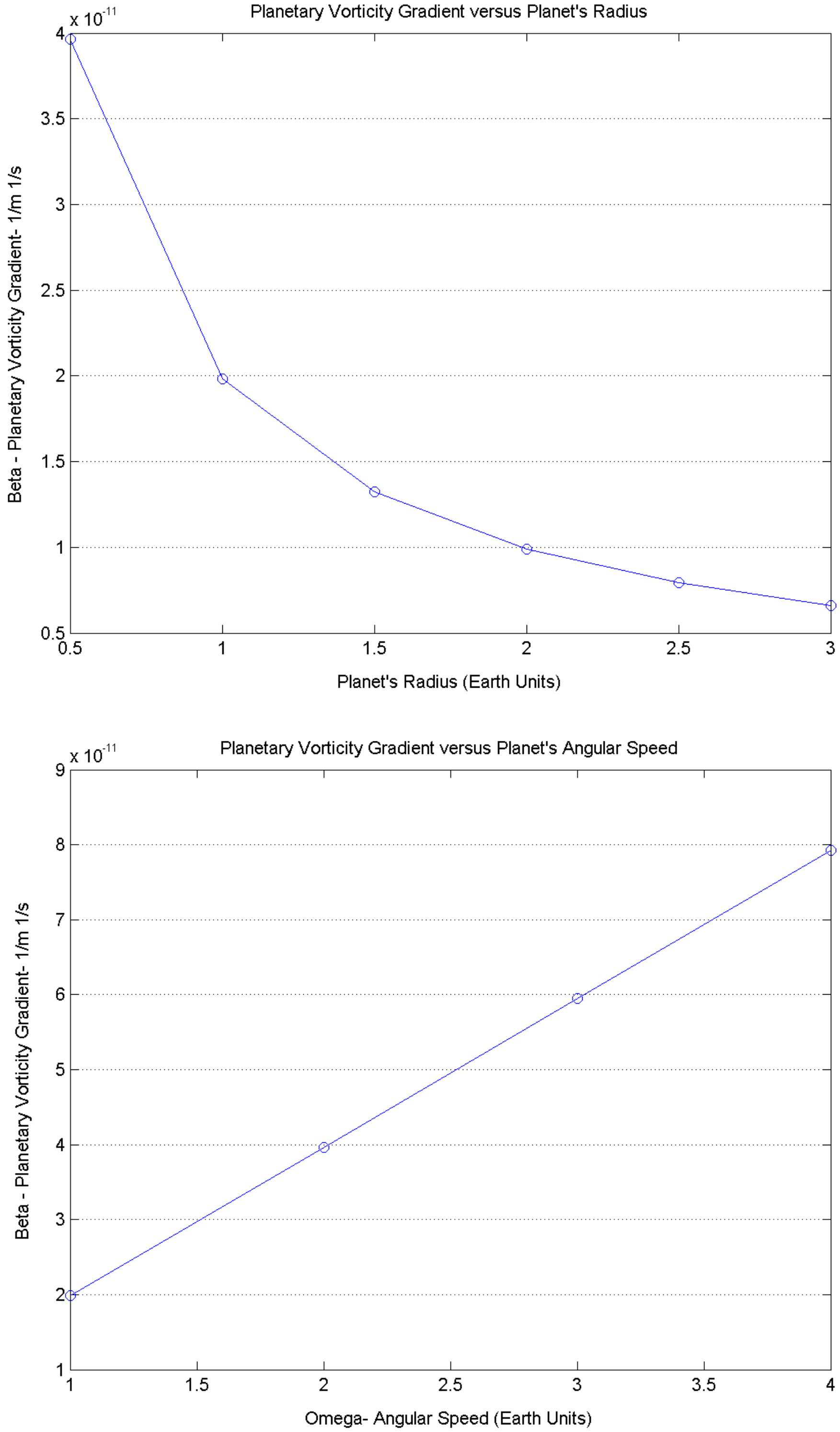
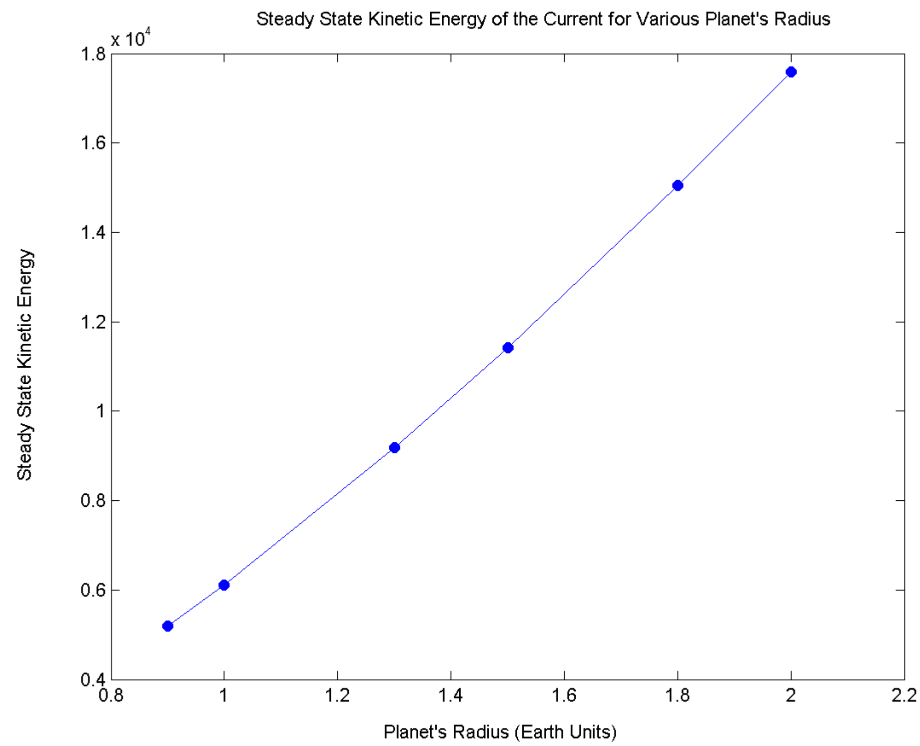


Figure 2. (TOP) Planetary Vorticity Gradient, β, for various planet radiuses. (BOTTOM) Planetary Vorticity Gradient, β, for various angular velocities, Ω.



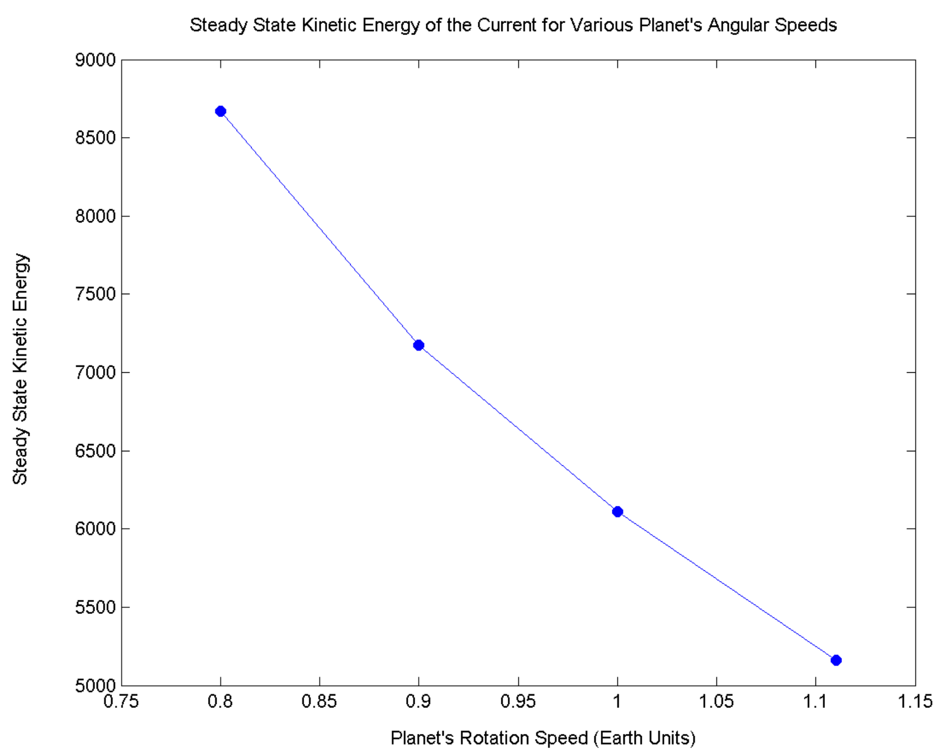


Figure 3. (TOP) Steady state total kinetic energy of the ocean current (units, $m^3 s^{-2}$) for various planet radiuses. (BOTTOM) Steady state total kinetic energy of the ocean current for various angular velocities.

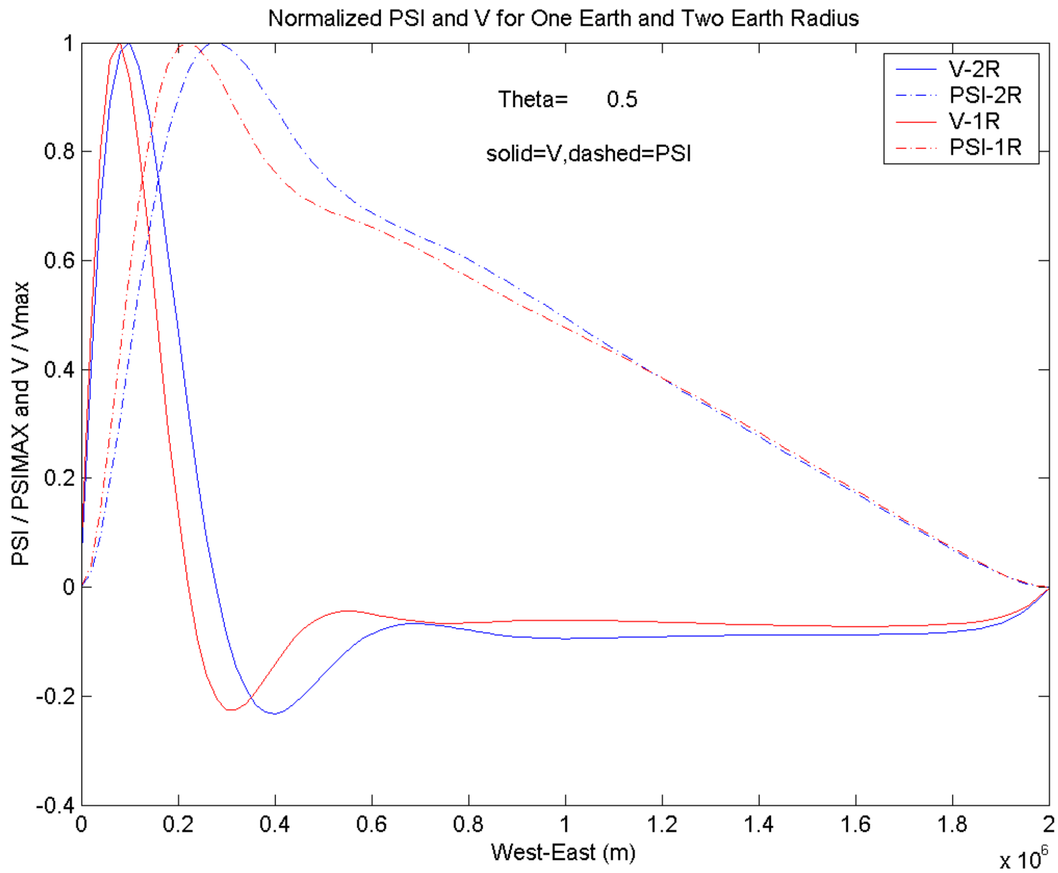


Figure 4. Comparison of model output: normalized stream function ψ, PSI, and normalized meridional current, *v*; for two particular cases: one earth radius (red curve) and two earth radius (blue curve).

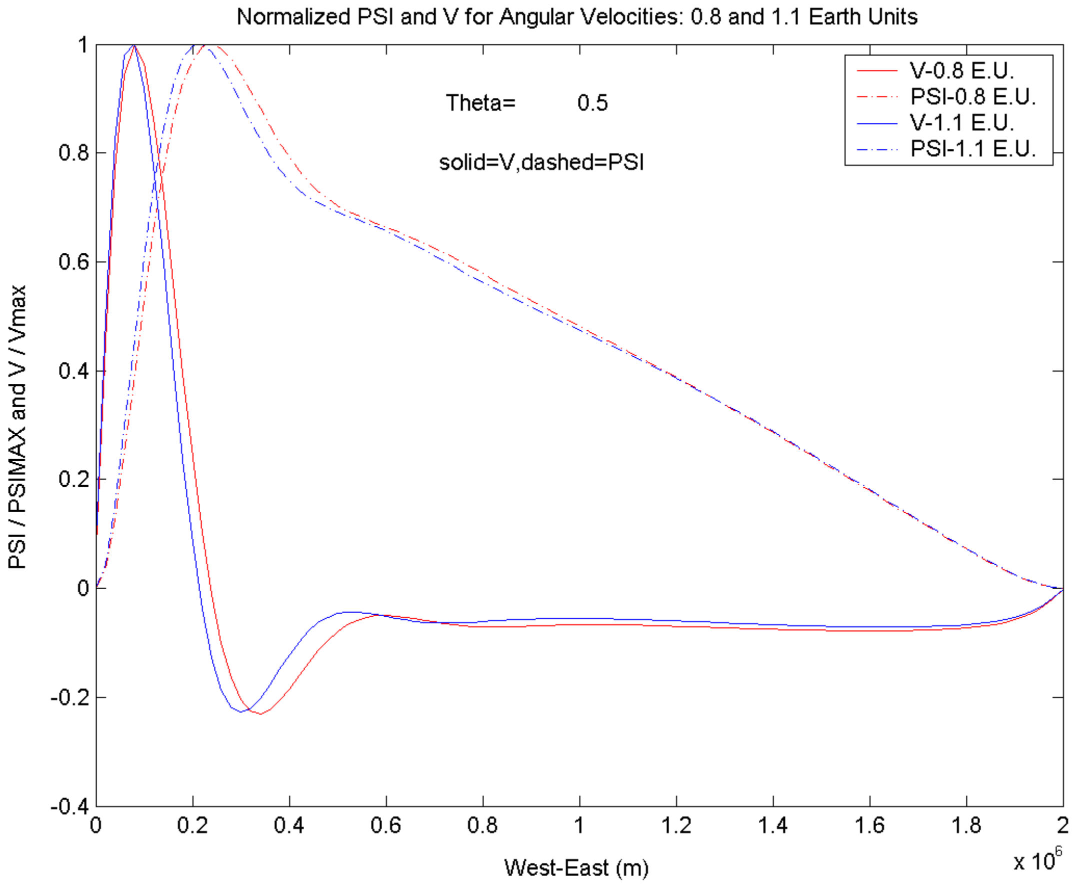


Figure 5. Comparison of model output: normalized stream function ψ, PSI, and normalized meridional current, *v*; for two angular speeds expressed in Earth Units: $0.8\Omega_0$ (red curve) and $1.1\Omega_0$ (blue curve).

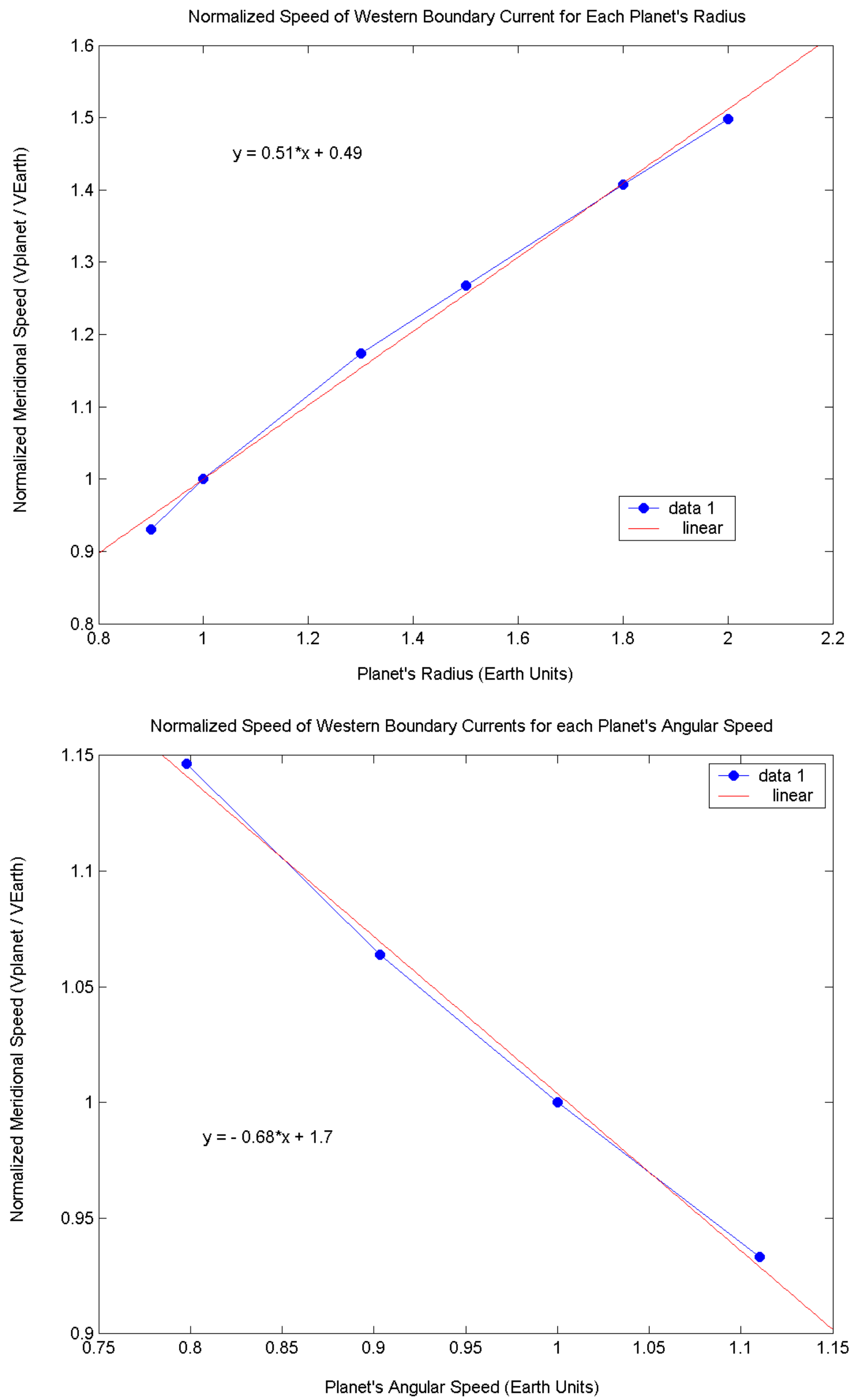
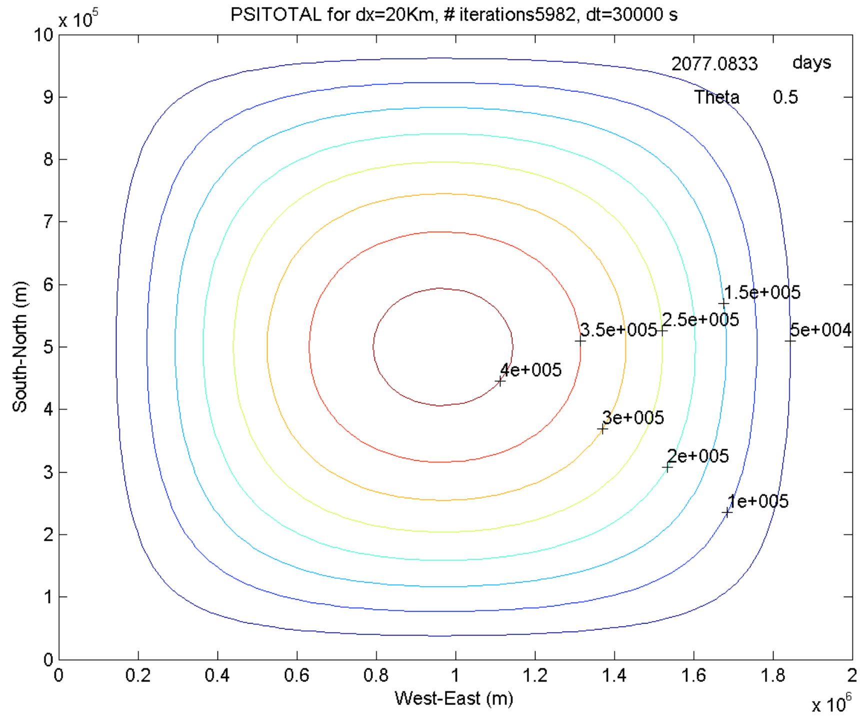


Figure 6. (TOP) Normalized meridional speed of the western boundary current for various planet radiuses. (BOTTOM) Normalized meridional speed, v, of the western boundary current for various angular velocities.



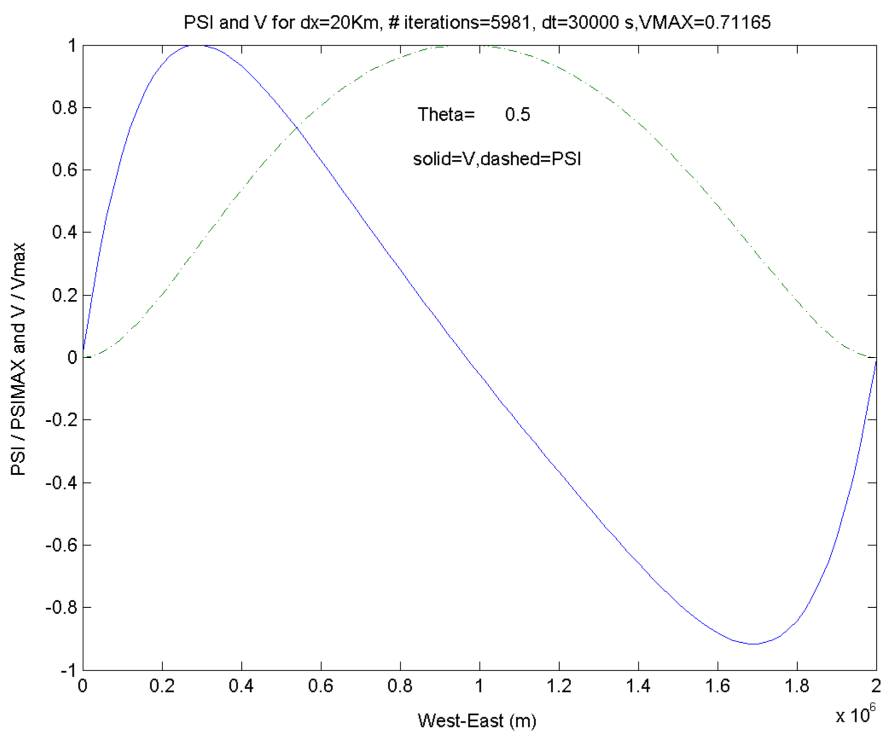


Figure 7. (TOP). Flow pattern (streamlines) for a tidally-locked Earth with a rotation period equal to one sidereal year. Note the absence of westward intensification. (BOTTOM) Normalized stream function ψ, PSI, and normalized meridional current, *v*, for an angular speed equal to $0.0027\Omega_0$. Note the maximum northward (pos.) and southward (neg.) currents show almost equal speeds.