Postdoctoral Fellowship Final Report

1. Title: **Response of baroclinic tides to the spring-neap cycle of the barotropic forcing**

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4. International Posts/Travel: None

5. Lectures:

   a. Seminar titled: Response of the nonlinear-nonstationary baroclinic tide to the spring-neap cycle of the barotropic tide. Thursday 14 November 2002. Code 7300 Division Conference Room.
   b. Poster presentation at the TOS-OIA 2003 Ocean Conference, Ernest N. Morial Convention Center, New Orleans, June 4-6, 2003.

6. Patents/Publications:
   a. The Hilbert-Huang Empirical Orthogonal Function and its application to the nonlinear-nonstationary baroclinic tide (In preparation)
   b. Coastal seiches, baroclinic tide generation, and diapycnal mixing off Puerto Rico (working on received peer reviews)
   c. Baroclinic tide-induced variations in primary productivity and optical properties in the Mona Passage, Puerto Rico (working on received peer reviews)

7. Research
   a. Long Term Goal: The long-term goal of our research is to identify and quantify the response of coastal seiches, baroclinic tides and waves to the spring-neap cycle of barotropic tidal forcing. This knowledge will make possible to predict when the large events will occur and reduce the risk of damage to vessels, people and the ecosystem.

b. Objective: The focus of this study is to understand the dependence of baroclinic tidal energy on the spring-neap cycle of barotropic forcing. Another key objective is the development of tools for analysis of nonlinear and nonstationary geophysical processes such as the baroclinic tides. These new methods will be applied to estimate the response of the nonlinear nonstationary baroclinic tide to the spring-neap cycle of the barotropic tide.

c. Approach: In fall 2002 an extensive set of oceanographic measurements were made by investigators at the Naval Research Laboratory (NRL) in a region of the Mid-Atlantic Bight near 39.3 N, 72.7 W (Figure 1). The observations were made in support of the Navy's Shallow Water Acoustics Experiment. Positions of moored ADCP's deployed from 30 September to 7 November, 2000 appear in Figure 1. We focused in analyzing the data from the stations: ADCP1, ADCP2, ADCP3. These locations allow us to see simultaneously the response of the baroclinic tide in shelf waters (~ 80 meters). Understanding the role of the barotropic tide in the generation of large baroclinic tides during neap tides is required. Baroclinic tides are a nonlinear-nonstationary baroclinic process generated by the interaction of the barotropic tide with the shelf break or slope topography. Dissipation of neap generated large-amplitude baroclinic tides could be related to the observed increase in nonlinear baroclinic waves (NIW's) and large amplitude coastal seiches. One of the challenges of studying baroclinic tides is their transient nature. These properties make traditional oceanographic analysis cumbersome. Fourier and wavelet analysis provide incomplete views of the phenomenon. New approaches are necessary to examine this type of geophysical signal.

   To study the relationship between baroclinic tides and coastal seiches, the Department of Marine Sciences of the University of Puerto Rico deployed two ADCP south of la Parguera, Puerto Rico near 17.863 N, 67.0383 W (Figure1B). The 76 kHz broadband ADCP was deployed off the insular shelf at a depth of 500 m from March 21 2002 to May 8 2002. Valid measurements were confined to a depth range from 27.4 m down to 452.4 m. The sampling time was six minutes. The second 150 KHz ADCP was deployed on the shelf at a depth of 23 m and about 100 m from the shelfbreak. This instrument took valid measurements from 5 to 21 m depth with the same six minute sampling rate. The record started on March 21 2002 and ended on April 5 2002. Concurrent measurements were obtained for about 15 days.

d. Work completed: We completed the analysis of the ADCP data from the three stations off the coast of New Jersey and from the two stations offshore La Parguera, Puerto Rico. We applied HEOF and more traditional analysis methods such as: complex demodulation and harmonic analysis. The same methodology was followed to analyze the data of New Jersey and Puerto Rico. The EMD analysis was corroborated with more standard analysis such as Fast Fourier Transform (FFT) and Discrete Wavelet Transform (DWT).

e. Research Objective & Impact: Our objective is to improve predictability of occurrence of baroclinic tides and higher frequency baroclinic waves, bores and solitons. Such phenomena strongly influence acoustic variability.

Primarily, a link with predictable barotropic tidal forcing is sought, but it is clear that better information on stratification is essential.

Novel techniques have been developed for the analysis of measurements of inherently nonstationary, nonlinear hydrodynamic processes. These include the HEOF method. HEOF analysis has proven useful in the analysis of baroclinic tides and its use could be extended to other nonlinear geophysical time series associated with spatially coherent modal structures.

1. Analysis

Resampling

The eastward (U) and northward (V) currents were low-pass filtered (to avoid aliasing) and resampled at a one hour interval. High frequency baroclinic waves were thus eliminated. This time interval is adequate for analysis of the baroclinic tide.

Removal of the barotropic tides: harmonic analysis

Harmonic Analysis (HA) was applied to the velocity series individually for each depth. We applied a least squares method (Emery and Thomson 2001) to fit 16 constituents: M_2, K_1, O_1, S_2, MSf, N_2, Q_1, Mm including the shallow water constituents (MK_3, SK_3, M_4, MS_4, S_4, $2MK_5$, $2SK_5$, M_6), which are resolvable according to the Rayleigh criterion. The M_2 constituent dominates the barotropic signal; for example, at 30 m the M_2 amplitude is 10.4 cm s^{-1} followed by Mm, MSf with 3.8 cm s^{-1} and 3.6 cm s^{-1}, respectively. The dominant shallow water constituent is MS_4 with 0.5 cm s^{-1}. The mooring velocity time series was detided by subtracting the least squares fitted signal at each depth level. Hence, in the diurnal and semidiurnal band, this residual consists primarily of baroclinic tidal currents. The least squares fitted signals were vertically averaged to define the barotropic spring-neap cycle at the station.

Removal of near-inertial oscillations: Bandpass Filter

Energy at and near the local inertial frequency (f=1.27 CPD) was removed from the detided time series using a second order Butterworth Infinite Impulse Response (IIR) bandpass filter. The Butterworth filter guaranteed zero phase shifts and was designed carefully to keep short the span of the impulse response. The filter passband was chosen to isolate those frequencies between 1.1 and 1.4 CPD. This signal was subtracted from the detided signal at each depth level. What remains is a signal that consists mostly of baroclinic tidal and subtidal fluctuations.

Removal of terdiurnal and fourth diurnal oscillations: Low Pass Filter

Energy at frequencies higher than 2.6 CPD was removed from the detided inertial-removed currents. A carefully designed symmetric Butterworth filter of order three guaranteed zero phase distortion and minimized edge effects on the time series. The remaining signal, dominated by the semidiurnal baroclinic tide, was used in the HHT analysis.

Hilbert-Huang Transform (HHT) Analysis

In this paper we provide only a brief explanation of HHT analysis. It consists of a combination of Empirical Mode Decomposition (EMD) and Hilbert Spectrum Analysis (HSA). A detailed explanation can be found in Huang et al. (1998, 1999). The application of HHT for analyzing nonstationary and nonlinear geophysical series has been reported in a number of papers (Huang et al. 2001, Huang et al. 2000, Zhu et al. 1997). HHT performs time-frequency analysis of nonstationary signals, as do other techniques such as wavelets (Huang et al. 1998). However, HHT is better suited than wavelets for analyzing signals resulting from nonlinear processes. This advantage motivated us to choose HHT to analyze our data. EMD can separate the signal into orthogonal components with different time scales.

HHT analysis starts with empirical mode decomposition (EMD). This technique decomposes time series data into a finite number of intrinsic mode functions (IMFs) with time variable amplitudes and frequencies. The decomposition is orthogonal and adaptive[1]. Any function is an IMF if (1) in the whole data set, the number of extrema and the number of zero-crossings is either equal of differ at most by one, and (2) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima are zero. The second criterion means that the function has symmetric envelopes defined by local maxima and minima respectively. The process to achieve this decomposition is called "extrema sifting". We limit the sifting to 100 times with a stoppage criterion of 5 times (CE(100,5), in the notation used by Huang, et al., 1998). This method allows us to (e.g.) easily separate the baroclinic tide from the subtidal components (Figure 2). We focus on the first IMF, F_1, which represents the semidiurnal

[1] By adaptive we mean that the EMD decomposition adapts to the local variations of the data. Adaptive basis is indispensable for nonstationary and nonlinear data analysis.

baroclinic currents. An IMF is a band-limited signal which is amenable to the Hilbert transform. Hence, each IMF component can be expressed as an analytic signal:

$$c(t) = a(t)\exp(iq\,(t))\,, \tag{1}$$

where $a(t)$ is the time varying amplitude and $q\,(t)$ is the time varying phase. These can be obtained from

$$a(t) = [X^2(t) + Y^2(t)]^{1/2}, \qquad q\,(t) = \arctan\left(\frac{Y(t)}{X(t)}\right) \tag{2}$$

X(t) and Y(t) represent the real and imaginary part of c(t), respectively. The imaginary part was defined by the Hilbert transform of the original signal, X(t).

$$Y(t) = \frac{1}{p} P \int_{-\infty}^{\infty} \frac{X(t')}{t - t'} dt', \tag{3}$$

where P indicates the Cauchy principal value. Y(t) is a version of the original real sequence with a 90° phase shift.

Variable $a(t)$ is the time varying amplitude of the analytic signal *c(t)*. It traces the envelope amplitude of the IMF component (Figure 3). $a^2(t)$ is then the relative energy of the modulation envelope.

The instantaneous frequency is defined as the time derivative of the instantaneous phase angle $q\,(t)$

$$w = \frac{dq}{dt}. \tag{4}$$

Once we have decomposed the original series *X(t)* into *L* IMFs and applied the Hilbert transform to each, we can represent the original series as

$$X(t) = \sum_{\ell=1}^{L} c_\ell(t) = \sum_{\ell=1}^{L} a_\ell(t)\exp(i\int w_\ell(t)\,dt). \tag{5}$$

Using (1) and (4), we define the Hilbert amplitude spectrum as, $H_\ell(w_\ell(t), t) = a_\ell(t)$ at $w_\ell(t)$, for all modes ℓ. *H* is displayed as a contour plot in the time-frequency plane in Figure 8A. Alternatively, using the square of *a(t)* results in the Hilbert energy spectrum.

The foregoing is a synopsis of the HHT method. Details can be found in the cited references. In the remainder of this paper we discuss combined HHT and EOF analysis, as applied to the ADCP records.

Hilbert-Huang Empirical Orthogonal Function (HEOF) Analysis

First Approach

ADCP data consists of velocity time series at discrete depth levels. We represent the data as a matrix **X** with K rows and M columns (matrices are denoted with a bold font),

$$\mathrm{X} = X(t_k, z_m) = \begin{bmatrix} X(t_1,z_1) & X(t_1,z_2) & X(t_1,z_3) & \cdots & X(t_1,z_M) \\ X(t_2,z_1) & X(t_2,z_2) & X(t_2,z_3) & \cdots & X(t_2,z_M) \\ X(t_3,z_1) & X(t_3,z_2) & X(t_3,z_3) & \cdots & X(t_3,z_M) \\ \vdots & \vdots & \vdots & & \vdots \\ X(t_K,z_1) & X(t_K,z_2) & X(t_K,z_3) & \cdots & X(t_K,z_M) \end{bmatrix} \tag{6}$$

Let $X(t_k, z_m)$ denote the value of any variable at depth z_m (i=1, 2 ,3…M) and time t_k (k=1, 2, 3…K). Each column is a scalar time series of K observations for a given depth. The rows are vertical profiles; they represent the observations at M locations for time t_k (k=1, 2, 3…K). In our particular case each column is a time series of baroclinic speeds u (E-W speed component) for each depth level, z_m (i=1, 2, 3…M). Applying EMD decomposition on the first column of matrix **X**, generates L IMFs: $F_1(t_k,z_1)$, $F_2(t_k,z_1)$, $F_3(t_k,z_1)$… $F_L(t_k,z_1)$ (k=1, 2, 3…K) for the first depth, z_1. Repeating this for the rest of the columns (z_1, z_2, z_3 … z_M) results in a 3D matrix $\mathbf{F} = \mathbf{F}_\ell(t_k, z_m)$ (i=1, 2, 3… M) (k=1, 2, 3…K) (ℓ=1, 2, 3…L), where L is the total number of IMFs at each depth z_m (Figure 4).

We decompose the new 3D matrix **F** into a series of 2D matrices, slicing it at a particular component ℓ (Figure 5). $\mathbf{F}_\ell$ has the dimensions K x M and its structure is similar to **X**. $\mathbf{F}_\ell$ for ℓ=1 corresponds to the semidiurnal time scale in our data.

$$\mathrm{F}_l = \mathrm{F}_l(\mathrm{t_k}, \mathrm{z_m}) = \begin{bmatrix} \mathrm{F}_l(\mathrm{t_1},\mathrm{z_1}) & \mathrm{F}_l(\mathrm{t_1},\mathrm{z_2}) & \mathrm{F}_l(\mathrm{t_1},\mathrm{z_3}) & \cdots & \mathrm{F}_l(\mathrm{t_1},\mathrm{z_M}) \\ \mathrm{F}_l(\mathrm{t_2},\mathrm{z_1}) & \mathrm{F}_l(\mathrm{t_2},\mathrm{z_2}) & \mathrm{F}_l(\mathrm{t_2},\mathrm{z_3}) & \cdots & \mathrm{F}_l(\mathrm{t_2},\mathrm{z_M}) \\ \mathrm{F}_l(\mathrm{t_3},\mathrm{z_1}) & \mathrm{F}_l(\mathrm{t_2},\mathrm{z_2}) & \mathrm{F}_l(\mathrm{t_3},\mathrm{z_3}) & \cdots & \mathrm{F}_l(\mathrm{t_3},\mathrm{z_M}) \\ \vdots & \vdots & \vdots & & \vdots \\ \mathrm{F}_l(\mathrm{t_K},\mathrm{z_1}) & \mathrm{F}_l(\mathrm{t_K},\mathrm{z_2}) & \mathrm{F}_l(\mathrm{t_K},\mathrm{z_3}) & \cdots & \mathrm{F}_l(\mathrm{t_K},\mathrm{z_M}) \end{bmatrix} \tag{7}$$

The covariance matrix of $\mathbf{F}_{\boldsymbol{l}}$ (between depths z_i and z_j) is

$$R_l(z_i, z_j) = \frac{1}{K}\sum_{k=1}^{K} F_l(t_k, z_i) F_l(t_k, z_j)$$
where $i, j = 1, 2, 3 \cdots M$ depths. (8)

Following Kundu et al. (1975), we find the eigenvectors and eigenvalues of the covariance matrix $R_l(z_i,z_j)$,

$$\sum_{i=1}^{M} R_l(z_i, z_j) f_n(z_i) = ?_n f_n(z_j), \quad n = 1, 2, 3 \cdots M \tag{9}$$

The eigenvectors $f_n(z_i)$ are orthonormal,

$$\sum_{j=1}^{M} f_n(z_j) f_m(z_j) = d_{nm} \tag{10}$$

Here, $F_l\ (t_k, z_i)$ may be expanded by $f_n(z_i)$, such that

$$F_l(t_k, z_i) = \sum_{n=1}^{M} E_n(t_k) f_n(z_i) \tag{11}$$

$$E_n(t_k) = \sum_{i=1}^{M} F_l(t_k, z_i) f_n(z_i), \tag{12}$$

where the $E_n(t_k)$ are orthogonal and satisfy,

$$\sum_{j=1}^{M} E_n(t_k) E_m(t_k) = l_m d_{nm}, \tag{13}$$

and $?_m$ is the total variance of mode *m*.

The $f_n(z_i)$, are the EOFs and the $E_n(t_k)$ are the principal components (PCs) of the expansion (11).

Following the above procedure (8-13) we obtain an EOF decomposition for each IMF, ℓ, resulting in a 3-D matrix (KxMxL) for each empirical mode; i.e. F^n, n=1,2,3…,M (see Figure 6). For IMF mode ℓ, the first 6 empirical modes explain 93.5% of the variance; Table 1 shows the distribution of variances in these modes.

The first IMF mode F_1^n (the semidiurnal band) EOFs and PCs are compared to dynamical modes (Kundu et al., 1975) based on a nearby buoyancy frequency profile. Correlations were high (>0.99 for mode 1 and >0.82 for mode 2) indicating that EOF modes are consistent with dynamical modes for the F_1 series for at least the first two empirical modes.

Applying equations 1-5 to matrix $\mathbf{F}^n$ yields the Hilbert spectrum for each depth and particular EOF mode, $\mathbf{H}^n \equiv H_\ell^n(w_\ell(t_k), t_k, z_i)$ (i=1, 2, 3… M) (k=1, 2, 3…K) (ℓ=1, 2, 3…L). This procedure, in contrast with the direct application of HSA to matrix **F,** reveals the specific time-frequency-depth patterns in each mode. This decomposition process is henceforth referred to as HEOF expansion.

Equation 7.3 from Huang et al. (1998), using our total data length, T = 39.6 d, and sampling rate, ? t = 0.0417 d, limits the maximum number of frequency cells to 188. The lowest frequency we can extract from the data is 1/T = 0.0253 CPD.

Examination of temperature data from the moored thermistor chain at the ADCP-3 location shows that the pycnocline depth oscillates between 30-40 meters. Hence, for a detailed look into the semidiurnal frequency band, we examine the Hilbert amplitude spectrum at 30 meters for the first three modes (n= 1, 2, 3); $H_1^n(?_1(t_k), t_k, 30)$ is plotted in Figures 7B-D. In the first mode, frequencies are confined to a band limited by 0.6 and 3.0 CPD, but high amplitude events are in a narrower band between 1.5 and 2.3 CPD. Higher energy is evident on YD 281.9, 290.9 and YD 303.6. The second mode shows high amplitudes on YD 276.9, 284.6, 292.0, 303.4, 309.8, especially on YD 296.6. The third mode shows similar spread in frequencies, but less energy. Maximum energy occurs on YD 278.0.

Figure 8 shows a time lag of 1-2 days (i.e. age of the tide) between the time of spring barotropic currents and the time of maximum astronomical forcing during syzygy. The maximum range in barotropic currents occurred on YD 273.57, YD 288.59, YD 300.52. Based on these numbers, the observed age of the barotropic tide oscillated from -0.81 days up to 1.74 days (Table 2). The age of the tide determined from the age of phase inequality between the M_2 and S_2 phases obtained from HA of the currents is 0.81 days.

The semidiurnal IMF component matrix, $\mathbf{F_1}$ and the corresponding EOF modes: $\mathbf{F_1^1}$, $\mathbf{F_1^2}$, $\mathbf{F_1^3}$ are shown in Figure 9A-D. Intensification of the first EOF mode baroclinic currents occur around YD 274.0, 282.2, 291.0, 298.5, 303.6 and, YD 311.6 (Figure 9B). These events are separated from each other by 8.2, 8.8, 7.5, 5.1, 8 days, respectively. The separation times average 7.5 ± 1.4 days. The peak baroclinic currents occur between 0.4 to 3.0 days after the barotropic spring currents. But stronger baroclinic tides follow barotropic neap tides by 2.4-3.6 days. This is more obvious in the second EOF mode which shows strong currents between 30-50 m on YD 296, one day after neap currents. (Figure 9C). The time lag of the peak baroclinic currents relative to the barotropic spring currents suggests a 0.4-3 days travel from the generation area of the baroclinic tide (Figure 10).

Second approach

In the first approach we examine the Hilbert Amplitude Spectrum for each empirical mode, empirical n-mode at each depth. An alternative approach is to construct a matrix, **A**, from amplitude, $a(t)$, of the analytical signal, c(t), derived from the Hilbert transform (Eq. 1-5) of each IMF component of the original data series X(t,z). Matrix **A** has dimensions (K x M x L), where L is the maximum number of IMF components.

$$\mathrm{A}_{\ell} = a_{\ell}(t_k, z_m) = \begin{bmatrix} a_{\ell}(t_1,z_1) & a_{\ell}(t_1,z_2) & a_{\ell}(t_1,z_3) & \cdots & a_{\ell}(t_1,z_M) \\ a_{\ell}(t_2,z_1) & a_{\ell}(t_2,z_2) & a_{\ell}(t_2,z_3) & \cdots & a_{\ell}(t_2,z_M) \\ a_{\ell}(t_3,z_1) & a_{\ell}(t_3,z_2) & a_{\ell}(t_3,z_3) & \cdots & a_{\ell}(t_3,z_M) \\ \vdots & \vdots & \vdots & & \vdots \\ a_{\ell}(t_K,z_1) & a_{\ell}(t_K,z_2) & a_{\ell}(t_K,z_3) & \cdots & a_{\ell}(t_K,z_M) \end{bmatrix} \tag{14}$$

where $a_{\ell}(t_k, z_m)$ (ℓ=1, 2, 3…L) (k=1, 2, 3…K) (m=1, 2, 3… M) is the amplitude of the analytical signal, c(t), for the chosen IMF component, ℓ.

As was done for matrix, **F** we slice the 3D matrix **A** along each IMF component, ℓ, and calculate EOFs and PCs.

In this case the covariance matrix of $\mathbf{A}_{\ell}$ is

$$R_{\ell}(z_i, z_j) = \frac{1}{K}\sum_{k=1}^{K} a_{\ell}(t_k, z_i)\, a_{\ell}(t_k, z_j)$$
$$\text{where } i, j = 1, 2, 3 \cdots M \quad \text{depths} \tag{15}$$

Analogous to equation 9, 11 and 12 we obtain the eigenvectors and eigenvalues from the covariance matrix and $a_{\ell}(t_k, z_i)$ can be expanded by $f_n(z_i)$,

$$a_{\ell}(t_k, z_i) = \sum_{n=1}^{M} E_n(t_k)\, f_n(z_i) \tag{16}$$

where

$$E_n(t_k) = \sum_{i=1}^{M} a_{\ell}(t_k, z_i)\, f_n(z_i) \tag{17}$$

Again, this procedure yields a set of 3-D matrices $\mathbf{A^n}$ (n=1, 2, 3…M) for the empirical modes. For ℓ=1, the first 6 modes explain about 88.4% of the variance (Table 1). We plot the results for $\mathbf{A_1^n}$ (n=1, 2, 3) in Figure 11B-D. It shows the time-space distribution of the amplitude of the envelope that modulates the semidiurnal baroclinic tide. The redder (bluer) areas indicate maximum (minimum) amplitude modulation.

Figure 11A shows matrix $\mathbf{A_1}$ (before EOF decomposition). Currents above 6 cm/s are prominent at 10-40 m including the pycnocline region (30-40 m). In addition strong currents are limited to the pycnocline (~30 m) on YD 296. After EOF analysis, the first mode, $\mathbf{A}_1^1$, is prominent on YD 273.8, 282.1, 291.0, 298.7, 303.6 and YD 311.6 (Figure 11B). These large amplitude events are separated from each other by 8.3, 8.9, 7.7, 4.9, 8.0 days, respectively. This result suggests that the baroclinic tidal amplitude has a mean timescale of 7.6 ± 1.6 days, half the time scale of the barotropic spring-neap cycle. This explains why large amplitude modulation (redder areas) on YD 282.1, 298.7 and 311.6 may follow the barotropic neap tides by a few days (Figure 12). Large amplitude modulation occurs 2.25, 3.83, and 2.74 days after the barotropic neap tides (Table 2). Large amplitudes are also seen in the second and third EOF modes (Figure 11C and 11D). These results demonstrate that large amplitude baroclinic tides may occur after barotropic neap tides.

To provide an estimate of the fluctuation time scale of baroclinic tidal amplitude we further decompose each series in $\mathbf{A_1}$ into IMFs, to allow a quantitative estimate of modulation frequencies. Here, the sifting results in only five IMFs. The resulting Hilbert amplitude spectrum (Figure 13A) traces (for example) the modulation envelope of $F_1(t_k,30m)$. The resulting Hilbert spectrum shows a strong fortnightly component in the modulation (Figure 13A), with the fortnightly cycle contained mainly in the last IMF. By selecting and grouping the instantaneous frequencies with nonzero amplitudes (values > 0.05) we compute an empirical probability density estimate (PDE) of $?_1(t)$ for each depth. The kernel density estimation is based on optimal normal bandwidth (Bowman and Azzalini, 1997). Merging all of them into one plot helps define the time scale of the baroclinic tidal amplitude (Figure 13C). Most of the peaks are concentrated between 0.05 CPD and 0.1 CPD (20-10 days). The average of all the PDE's (traced by the thick line) shows a peak centered on 0.07 CPD (14 days). The strong fortnightly-scale may obscure the other time scales of modulation. If we remove the last IMF this problem can be overcome. The new Hilbert spectrum is shown in Figure 13B. The new PDE's shift to the right showing most peaks between 0.1 CPD and 0.2 CPD (Figure 13D). These peaks correspond to modulations of the semidiurnal baroclinic tide ranging between 5-10 days. The average of all PDE's shows a modulation frequency of 0.135 CPD (7.4 days). These results show that the fluctuation of baroclinic tidal amplitude does not show a regular 2-week cycle but instead a variable one.

Summary and Conclusions

Baroclinic tides in shelf waters are studied using data collected from one ADCP in the Mid-Atlantic Bight. The response of the baroclinic currents to the spring-neap cycle of the barotropic currents is explored using two approaches of HEOF analysis. Both approaches reduce complexity of HHT analysis of the ADCP data.

HEOF analysis is based on modal decompositions of a covariance matrix calculated from the EMD analysis of the original series. We have shown two ways to implement it: the first approach consisted of EMD, EOF, HSA and the second approach of EMD, Hilbert transform, EOF, in that respective order. The first allows us to see the Hilbert amplitude spectrum for selected empirical modes and the second shows the amplitude of the intrinsic mode function's envelope for each empirical mode. One is a complement of the other so both approaches should be used to provide a complete analysis of the ADCP data.

This method allows the detection of spatially coherent modal structures of (e.g.) nonlinear baroclinic waves. The second approach is useful to describe the temporal evolution of the envelope modulating the amplitude of the waves. In addition, EOF analysis identifies the spatial coherence of this modulation process.

Both approaches reveal a variable baroclinic tidal amplitude (between 10-20 days) with an approximate time lag of 0.4-3 days respect the barotropic spring-neap cycle probably due to the distance from the generation area. In addition, a 5-10 day modulation cycle is present. The second and third EOF modes show increased baroclinic tide 0.4-3 days after barotropic neap tides. A baroclinic tidal amplitude cycle of half the barotropic spring-neap cycle has been reported on current measurements near the Canary Basin (Siedler and Paul, 1991). They found that the first dynamic mode shows time scales of the order of 5 days, similar to the time scales observed in $\mathbf{A_1}$. In addition, our findings support some observations of stronger mixing during neap tides rather than during spring barotropic tides (Rippeth and Inall, 2002; Alfonso, 2002). Also a 5-10 d baroclinic cycle could explain observations of large amplitude coastal seiche phenomena observed during neap tides without the necessity to invoke the hypothesis of far generated baroclinic waves (Giese 1982). Finally, there are alternative explanations for the concurrence of strong baroclinic signals and weak barotropic tides. Using a numerical model, Gerkema (2002) established that a baroclinic spring-neap cycle may have large phase shifts with respect to the barotropic spring-neap cycle due to the different trajectories of the M_2 and S_2 baroclinic beams and the local difference in phase between them at different locations. Gerkema findings can explain a phase shift in a 2-week baroclinic spring-neap cycle but it seems more difficult that it could explain a 5-10 day cycle.

2. Results: ADCP1 & ADCP2

Kinetic energy, KE=0.5 ? (u2+v2), was calculated from F1: u,v. Potential energy was derived from the KE by means of the ratio PE/KE = (? 2-f2) / (? 2+f2)= 0.4; (Gill, 1992). Figure 14 shows the Kinetic Energy of F1 at each of the moorings: ADCP1, ADCP2, ADCP3. More energetic baroclinic tides are observed in ADCP1. Focusing our attention on particular strong events (dashed ellipses) it is obvious that a significant time lag of the baroclinic tide energy can occur in relatively short distances (D < 30 km). Comparing the previous results with the barotropic spring-neap cycle (Figure 8), it is evident that equally energetic baroclinic tides can occur during barotropic neap tides as during spring tides.

3. EMD versus FFT and Wavelets

Figure 15 shows the plot of the original signal at a particular depth, 30.5 m (top) and the baroclinic tide extracted through a SD FFT bandpass filter, EMD and wavelets, respectively. The FFT output is too regular and smooth, and wavelet is much sharper. EMD is a compromise between the two.

Comparison of the KE of the baroclinic tides derived through Fast Fourier Transform (FFT) analysis, (DWT) Discrete Wavelet Transform analysis and EMD analysis are shown in Figure 16-17. EMD has a superior time-space resolution than FFT and the maximum energy values can increase up to 2.67 times. Wavelet has a superior time resolution and the maximum energy values can increase up to 2 times. But the space vertical resolution is not as good as in EMD.

4. Results from La Parguera, Puerto Rico

a. Offshelf ADCP

The barotropic tidal currents determined from HA are shown in Figure 18. The currents show a mixed tidal regime. The cycle alternates dominance between diurnal and semidiurnal currents. The maximum tidal currents range (~ 3 cm/s) occur on YD 86 and YD 116 (Figure 19) and minimum tidal range on YD 106 (values at the edges are not considered).

EMD analysis allowed us to extract the SD baroclinic tide, F_1, and the diurnal baroclinic tide, F_2 from current speed components u and v (Figure 20A-B). Stronger SD baroclinic tides occur on YD 103, 111, 115 and YD 121. Stronger diurnal tides occur on YD 93, 104, 111 and YD 121. The SD and diurnal events occur simultaneous on YD 104 and YD 111 (Figure 20B).

From the first approach of HEOF we obtained $\mathbf{F_1}$, $\mathbf{F_1^1}$, $\mathbf{F_1^2}$, $\mathbf{F_1^3}$, all are shown in Figure 21A-D. Currents stronger than 6 cm/s occur above 75 m and near 200 m on YD 89. The second and third EOF mode, $\mathbf{F_1^2}$, show speeds less than 5 cm/s. The first 6 modes explain about 78.5% of the variance (Table 3).

From the second approach of HEOF we obtained a set of 3-D matrices $\mathbf{A^n}$ (n=1, 2, 3…M) for the empirical modes. For ℓ=1, the first 6 modes explain about 66% of the variance (Table 3). We plot the results for $\mathbf{A_1^n}$ (n=1, 2, 3) in Figure 22B-D. It shows the time-space distribution of the amplitude of the envelope that modulates the semidiurnal baroclinic tide. The redder (bluer) areas indicate maximum (minimum) amplitude modulation.

Figure 22A shows matrix $\mathbf{A_1}$ (before EOF decomposition). Currents above 8 cm/s are prominent above 75 m. In addition strong currents occur at 200 m on YD 89. After EOF analysis, the first mode, $\mathbf{A_1^1}$, is prominent on YD 84.2, 87.5, 94.9, 103.7, 107.6, 115.4 and YD 122.3 (Figure 22B). These large amplitude events are separated from each other by 3.2, 7.5, 8.8, 3.9, 7.8 and 6.9 days, respectively. This result suggests that the baroclinic tidal amplitude has a mean timescale of 6.3 ± 2.2 days, half the time scale of the barotropic spring-neap cycle.

To provide an estimate of the fluctuation time scale of baroclinic tidal amplitude we further decompose each series in $\mathbf{A_1}$ into IMFs, to allow a quantitative estimate of modulation frequencies. Here, the sifting results into only five IMFs. The resulting Hilbert amplitude spectrum (Figure 23A) traces (for example) the modulation envelope of $F_1(t_k,27.4m)$. It shows more energy above 0.1 CPD (Figure 23A). By selecting and grouping the instantaneous frequencies with nonzero amplitudes (values > 0.05) we compute an empirical probability density estimate (PDE) of $?_1(t)$ for each depth. Merging all of them into one plot helps define the time scale of the baroclinic tidal amplitude (Figure 23C). The average of all the PDE's (traced by the starred line) shows a peak centered on 0.127 CPD (7.8 days). Repeating the same analysis but this time using $\mathbf{A_2}$ (i.e. modulations of the diurnal baroclinic tide). The new Hilbert spectrum is shown in Figure 23B. The Hilbert spectrum reveals that there is higher energy below 0.1 CPD. The new probability function shifts to the left and is centered on 0.062 (Figure 23D). This peak corresponds to modulations of 16 days. These results show that the fluctuation of baroclinic tidal amplitudes is mainly weekly for the semidiurnal baroclinic tide and fortnightly for the diurnal baroclinic tide.

The most energetic semidiurnal baroclinic tides occur on the top 75 meters. Kinetic energy (KE) and potential energy (PE) are shown in Figure 24A-B. Large KE occurs on YD 104, YD 108, YD 115 and YD 122. The diurnal baroclinic tides show large KE on YD 94, YD 96, and YD 98 (Figure 25B). Figure 26 shows an expanded view of the top 75 m. Energetic baroclinic tides equally occur at times of maximum and minimum tidal ranges. There is not a clear relationship between strong barotropic forcing and energetic baroclinic tides.

b. Shelf ADCP

The SD and diurnal baroclinic tide at 5 m, F_1 and F_2, respectively, show stronger currents between YD 90-92 (Figure 27A-B). Plotting F_1 from a depth of 5 m down to a depth of 14 m clearly shows the event on YD 91 (Figure 28A-B). The current speeds range between 6 cm/s and 10 cm/s (Figure 29A-B). The KE reach 3.0 J m^{-3} during this event (Figure 30A).

c. Magueyes Island Tidal Station, La Parguera

A NOAA tide station at La Parguera measures water level acoustically every six minutes. The tidal record from YD 79.33 to YD 127 is shown in Figure 31. The EMD of the tide record is shown in Figure 32. Components C2, C6 and C7 correspond to the seiche (T=50 min), the semidiurnal and diurnal tides, respectively. The largest component is the C7, the diurnal tide. Seiches are stronger from YD 93 to YD 97 and on YD 109 (enclosed by dashed ellipses). Strong seiches were observed during periods of strong barotropic diurnal tides.

d. Relationship between strong seiches and strong baroclinic tides.

Stronger seiches are observed between YD 93 up to YD 97 and on YD 109. The KE of the semidiurnal baroclinic tide on the shelf is high on YD 91 but the offshelf diurnal baroclinic tide is more energetic than the semidiurnal baroclinic tide during this particular period. In addition, the barotropic currents offshelf are mainly semidiurnal and medium range (~ 2.5 cm/s) but the offshelf baroclinic SD energy is low.

The seiche event on YD 109 corresponds to high SD baroclinic energy offshelf. The diurnal KE is low. The barotropic currents again are mainly semidiurnal and with a small range (< 2.2 cm/s).

It is interesting to see that at the times of predominantly barotropic semidiurnal currents offshelf, is when the tidal heights has its maximum diurnal range and seiches are stronger.

The above results imply that mainly barotropic semidiurnal currents are necessary for seiche generation.

5. References

Alfonso, E., 2002. *Variabilidad temporal de la produccion primaria fitoplanctonica en la estacion CaTS (Caribbean Time-Series Station): Con enfasis en el impacto de la marea interna semidiurna sobre la produccion.* Ph. D. Dissertation. Department of Marine Sciences, University of Puerto Rico, Mayagüez, Puerto Rico. UMI publication AAT 3042382, 407 pp.

Bowman, A. W., Azzalini, A. 1997. Applied Smoothing Techniques for Data Analysis, Oxford University Press, 208 pp.

Emery, W. J., and R. E. Thompson. 2001. Data analysis methods in physical oceanography. 2nd ed., Elsevier, New York, 638 pp.

Gerkema, T. 2002. Application of an internal tide generation model to baroclinic spring-neap cycles. Journal of Geophysical Research 107(C9), 3124.

Giese, G. S., Hollander, R.B., 1987. The relationship between coastal seiches at Palawan Island and Tide-Generated Internal Waves in the Sulu Sea. Journal of Geophysical Research 92(C5), 5151-5156

Giese, G. S., Hollander, R.B., Francher, J.E., Giese, B.S., 1982. Evidence of coastal seiche excitation by tide-generated internal solitary waves, Geophysical Research Letters 9, 1305-1308.

Huang, N. E., Chern, C. C., Huang, K., Salvino, L. W., Long, S. R. and K. L. Fan. 2001. A new spectral representation of earthquake date: Hilbert spectral analysis of station TCU129, Chi-Chi, Taiwan, 21 September 1999.

Huang, N. E., Shih, H. H., Shen, Z., Long, S. R., and K. L. Fan. 2000. The ages of large amplitude coastal seiches on the Caribbean Coast of Puerto Rico. Journal of Physical Oceanography 30(8), 2001-2012.

Huang, N. E., Shen, Z., and S. R. Long, M. C. Wu, H. H. Shih, Q. Zheng, N-C. Yen, C. C. Tung, and H. H. Liu. 1998. The empirical mode decomposition and the Hilbert spectrum fon nonlinear and non-stationary time series analysis, Proc. R. Soc. London Series A 454: 903-995.

Huang, N. E., Shen Z. and S. R. Long. 1999. A new view of nonlinear water waves: the Hilbert Spectrum, Annual Review of Fluid Dynamics 31:417-457.

Inall, M. E., Rippeth, T. P. and T. J. Sherwin. 2000. Impact of nonlinear waves on the dissipation of internal tidal energy at a shelf break. Journal of Geophysical Research 105(C4): 8687-8705.

Inall, M. E., Shapiro, G. I. and T. J. Sherwin. 2001. Mass transport by non-linear internal waves on the Malin Shelf. Continental Shelf Research 21: 1449-1472.

Kundu, P. K., J. S. Allen and R. L. Smith. 1975. Modal decomposition of the velocity field near the Oregon coast, J. Phys. Oceanogr., 5, 683-704.

Pineda, J. 1995. An internal tidal bore regime at nearshore stations along western U.S.A.: Predictable upwelling within the lunar cycle. Continental Shelf Research 15(8): 1023-1041.

Rippeth, T. P. and M. E. Inall, 2002. Observations of the internal tide and associated mixing across the Malin Shelf, Journal of Geophysical Research 107(C4), 3028.

Siedler, G., U. Paul. 1991. Barotropic and baroclinic tidal currents in the eastern basins of the North Atlantic. Journal of Geophysical Research 96(C12):22259-22271.

Wang J., Chern, C. C., and A. K. Liu. 2000. The wavelet empirical orthogonal function and its application to the analysis of internal tides. Journal of Atmospheric and Oceanic Technology 17: 1403-1420.

Zhu, X., Shen Z., Eckermann, S. D., Bittner, M., Hirota I., and J-H. Yee. 1997. Gravity wave characteristics in the middle atmoshphere derived from the Empirical Mode Decomposition method. Journal of Geophysical Research 102(D14):16545-16561.

Figures Captions

Figure 1. ADCP locations are indicated by triangles. Contours are in meters A moored thermistor chain was deployed at the ADCP-3 location for about 12 days.

Figure 2. IMF sifting of the one hour re-sampled baroclinic current, u(t), at 30 meters depth. F_1 shows a semidiurnal time scale and may represent the SD nonlinear baroclinic tide. Largest currents were observed on YD 296.

Figure 3. A closer look at the IMF component F_1 (for u(t)) corresponding to the semidiurnal baroclinic tide (continuous line). The analytic function, c(t), that results from the Hilbert transform traces the envelope of the SD baroclinic tide.

Figure 4. Diagram explains the generation of matrix **F** from the original data in matrix **X**. First, perform EMD analysis on the time series at each depth. This process generates L IMF's, for each of the M depth elevels. Second, group the results in the matrix **F.**

Figure 5. Decomposition of the 3D matrix **F** into a series of 2D matrices $\mathbf{F}_\ell$ slicing it at a each IMF component, ℓ, where ℓ=1, 2, 3…L. The dimension of $\mathbf{F}_\ell$ is K x M.

Figure 6. EOF decomposition of the 2D matrices: $\mathbf{F_1}$**,** $\mathbf{F_2}$**,** $\mathbf{F_3}$**…** $\mathbf{F_L}$.

Figure 7(A) Hilbert spectrum, $H(w_\ell(t_k), t_k, 30)$**,** before HEOF decomposition. Yellow-red patches inside the semidiurnal band (1.5-2.5 CPD) occurred on YD 286, 297 and 304 and indicates intensification of the SD baroclinic tide. **(B)** Hilbert spectrum, $H_1^1(w_\ell(t_k), t_k, 30)$ after HEOF decomposition using the first approach. **(C)** Same for mode two: $H_1^2(w_\ell(t_k), t_k, 30)$. **(D)** Same for mode three: $H_1^3(w_\ell(t_k), t_k, 30)$.

Figure 8. Barotropic spring-neap cycle versus the synodic cycle. The time interval between the lunar phase and the corresponding maximum (or minimum) range in tidal currents defines the age of the tide.

Figure 9.(A) Plot of $\mathbf{F_1}$, before HEOF decomposition. **(B)** Plot of $\mathbf{F_1^1}$ after HEOF decomposition using the first approach. **(C)** Same for mode two: $\mathbf{F_1^2}$**.** **(D)** Same for mode three: $\mathbf{F_1^3}$**.**

Figure 10.(A) Plot of $\mathbf{F_1^1}$ shows the semidiurnal baroclinic currents (u). **(B)** Barotropic spring-neap cycle.

Figure 11. (A) Plot of $\mathbf{A_1}$, before HEOF decomposition. **(B)** Plot of $\mathbf{A_1^1}$ after HEOF decomposition using the second approach. **(C)** Same for mode two: $\mathbf{A_1^2}$**.** **(D)** Same for mode three: $\mathbf{A_1^3}$**.**

Figure 12. (A) Plot of $\mathbf{A_1^1}$ shows the first EOF mode of the envelope $\mathbf{A_1}$**.** **(B)** Barotropic spring-neap cycle.

Figure 13. (A) Hilbert spectrum after applying EMD decomposition on the envelope, $\mathbf{A_1}$**.** **(B)** Same as figure 14A, but the IMF with fortnightly time scale was removed. **(C)** Empirical probability density functions (PDE's) of the envelope instantaneous frequencies for each depth. **(D)** Empirical probability density functions (PDE's) of the envelope instantaneous frequencies once the fortnightly-scale IMF was removed.

Figure 14. (A) KE in ADCP1. **(B)** KE in ADCP3. **(C)** KE in ADCP2. Dashed ellipses connect the same events of high energy baroclinic tides in each station.

Figure 15. Extraction of the SD baroclinic tide from the original signal (top) by a bandpass FFT filter (2nd plot), EMD (3rd plot) and wavelets (4th plot).

Figure 16. (A) KE of the SD baroclinic tide derived from FFT analysis. **(B)** KE of the SD baroclinic tide derived from EMD analysis.

Figure 17. (A) KE of the SD baroclinic tide derived from wavelet analysis. **(B)** KE of the SD baroclinic tide derived from EMD analysis.

Figure 18. Offshelf barotropic tidal currents (east-west component, U) obtained from harmonic analysis.

Figure 19. Range of the offshelf barotropic tidal currents (east-west component, U).

Figure 20. EMD sifting of the offshelf baroclinic semidiurnal currents (F_1). **(A)** Component u. **(B)** Component v.

Figure 21. Semidiurnal tide, F_1, offshelf La Parguera. **(A)** Plot of $\mathbf{F_1}$, before HEOF decomposition. **(B)** Plot of $\mathbf{F_1^1}$ after HEOF decomposition using the first approach. **(C)** Same for mode two: $\mathbf{F_1^2}$. **(D)** Same for mode three: $\mathbf{F_1^3}$.

Figure 22. (A) Plot of $\mathbf{A_1}$, before HEOF decomposition. **(B)** Plot of $\mathbf{A_1^1}$ after HEOF decomposition using the second approach. **(C)** Same for mode two: $\mathbf{A_1^2}$. **(D)** Same for mode three: $\mathbf{A_1^3}$.

Figure 23. (A) Hilbert spectrum after applying EMD decomposition on the envelope, $\mathbf{A_1}$. **(B)** Hilbert spectrum after applying EMD decomposition on the envelope, $\mathbf{A_2}$. **(C)** Empirical probability density functions (PDE's) of the envelope instantaneous frequencies for each depth, ($\mathbf{A_1}$) **(D)** Empirical probability density functions (PDE's) of the envelope instantaneous frequencies for $\mathbf{A_2}$.

Figure 24. (A) KE of the offshelf semidiurnal baroclinic tide. **(B)** PE of the offshelf semidiurnal baroclinic tide.

Figure 25. (A) KE of the offshelf diurnal baroclinic tide. **(B)** PE of the offshelf diurnal baroclinic tide.

Figure 26. (A) Expanded view the top 75 m of the KE of the SD baroclinic tide. **(B)** Expanded view of the top 75 m of the KE of the diurnal baroclinic tide.

Figure 27. EMD sifting of the shelf baroclinic semidiurnal currents (F_1) at 5 m depth. **(A)** Component u. **(B)** Component v.

Figure 28. Shelf baroclinic semidiurnal currents (F_1) in the top 14 meters. **(A)** Component u. **(B)** Component v.

Figure 29. Speed of the shelf baroclinic SD currents (F1). **(A)** Component u. **(B)** Component v.

Figure 30. (A) KE of the shelf semidiurnal baroclinic tide. **(B)** PE of the shelf semidiurnal baroclinic tide.

Figure 31. Tidal heights record from tide station located at Magueyes Islands, La Parguera, Puerto Rico.

Figure 32. EMD sifting of the tidal heights record. Extrema sifting, CE(113,5) and intermittency test was applied. C2 represents the barotropic seiche signal, C6 is the SD tide and C7 is the diurnal tide.