The 1.8-hours Sea-Level Oscillations Recorded at Isabel Segunda, Vieques and Puerto de Fajardo

By

Edwin Alfonso-Sosa, Ph.D.

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Abstract

Small sea level oscillations (amplitude ~ 0.1 feet = 3 cm) with time scales between 1.6 and 1.9 hours, and an average period (frequency) of 1.84 hours (13 CPD), have been recorded by two tide gauges located in two small harbors at Eastern Puerto Rico. The two harbors are connected by a shallow shelf with an average depth of 10 meters. The oscillation period is shorter than the twelfth-diurnal tidal constituents – with periods from 2.03 to 2.09 hours (see Table 1) – and longer than the sixteen-diurnal harmonic ($8M_2$ or M_{16}; period of 1.55 hours). It is possible that the period of the 1.8-hour oscillation is determined by the basin geometry and depth. In addition, it is an intermittent signal in the water-level record. The duration of the larger amplitude oscillations is less than 0.5 days. Maximum amplitudes reached up to 0.2 feet.

Site description

El Puerto de Fajardo is a small harbor, located in the Bay of Fajardo on the east end of the island of Puerto Rico. Fajardo Bay is bordered by shallow reefs and cays that protect it from short-period waves. The main entrance to the Bay is the NE, but has a narrow and shallow entrance in the direction ESE (Figure 1, arrow). It is the main port for transporting people from the big island of Puerto Rico to the islands of Culebra and Vieques. There are several marinas with docks for boats and yachts; it's one of the best places for maritime activity in Puerto Rico. Since October 2008, a tide gauge installed in tidal station 9753216 by NOAA / NOS / CO-OPS, began measuring every 6 minutes the water level in the harbor. About 28.6 kilometers SE of Puerto de Fajardo is located the Isabel Segunda Harbor, in the north side of Vieques Island (Figure 2, Top). These two harbors are connected by a shallow shelf with an average depth of 10 meters. The NOAA / NOS tidal station 9752619 located at Isabel Segunda have been recorded 6-minute water levels since March 2009. The tide in both harbors show a mixed semidiurnal character. The distance between the shelf break and Puerto de Fajardo is 50.77 km, making it the longest shelf of Puerto Rico, with an average depth of 25 meters (Figure 2, Bottom). Off the shelf break a steep slope descends into 400 m and 700 m depths.

Data Analysis

The 6-minute water level data was subjected to the Empirical Mode Decomposition (EMD), a novel method to analyze nonstationary and nonlinear signals (Huang et al. 1998). This adaptive method has been applied to the analysis nonlinear water waves (Huang et al. 1999). From EMD we obtain intrinsic mode functions (IMF) components that have a physical interpretation. Well known nonlinear systems such as the Duffing equation has been subject to Hilbert Huang Transform (HHT) analysis and each component have a physical meaning. Using Normalized Hilbert Transform (NHT) we found the instantaneous frequencies of each IMF component obtained by EMD. The concept of instantaneous frequency has been well explained by Huang et al. (2009).

This analysis was performed in Matlab, using the m-files provided by: 中央大學數據分析中 Research Center for Adaptive Data Analysis. Chungli,Taiwan http://rcada.ncu.edu.tw/intro.html

Results

Figures 3-5 and Figures 6-8 show the observed water level record, the predicted tide and the difference between the two series, for Isabel Segunda and Puerto de Fajardo, respectively. These figures show the 1.8-hour oscillation deforming the mixed semidiurnal tide form at both harbors. From a careful view at the figures it's clear that the 1.8-hour signal is intermittent in the water level record. Two events in Isabel Segunda reveal the non-stationary nature of the oscillation. On August 12 2003, the oscillation suddenly started at 03:30 GMT and lasted up to 10:30 GMT, about 0.29 days (Figure 3, Bottom). On September 7, 2013 the oscillation started just after 11:00 GMT and finished at 19:00 GMT, lasted about 0.33 days (Figure 4, Top). The third panel of Figure 15 shows the event of longer duration on October 25-27 2013, it started on YD 298.48 and ended on YD 300.18, about 1.7 days. The duration of most events was less than 0.5 days and their heights reached up to 0.4 feet around the perigean tides and near equatorial tides on July 18 and September 7, respectively (see table below).

Vieques-Fajardo Oscillation Date (GMT)	Event duration (hr)	Height (ft)	Astronomical Event Date (GMT)	Moon Dist (km)	Moon Dec.	Time Interval (d)	Tidal Constituent	Name of Tidal Constituent
6/19/2013 17:00	6	0.38	6/22/2013 23:20	354571		-3.26	N2	Large elliptical tide of first-order to M2
7/18/2013 18:00	8	0.40	7/21/2013 22:20	355544		-3.18	N2	Large elliptical tide of first-order to M2
8/12/2013 4:00	7	0.30	8/9/2013 16:20		0°6'	2.49	K1	Diurnal Principal Declination Tide
9/7/2013 11:00	8	0.40	9/5/2013 23:20		0°3'	1.49	K1	Diurnal Principal Declination Tide
10/25/2013 19:48	40.7	0.11		406258	16°16'			
5/8/2014 19:24	11	0.15	5/10/2014 0:20		0°0'	-1.21	K1	Diurnal Principal Declination Tide
								Ocean Physics Education Inc., 2014

Figures 9-10 show the first four intrinsic mode functions (IMF's) obtained from the empirical mode decomposition (EMD) of the sea level record of Vieques and Fajardo, respectively. The third IMF, C3, represents the 1.8-hour oscillation. An overlay of both C3's is shown in Figure 11. The large amplitude oscillations are practically in phase. A smaller event occurred on October 13-14 at both harbors (Figures 12-13). The overlay of the third IMF's (C3) shows that both sea level oscillations are in phase again (Figure 14). The longest oscillations occurred between October 25th and 27th and the EMD analysis reveals the five IMF's for that period (Figure 15). The fourth and fifth IMF (C4, C5) represent the semidiurnal and diurnal tides, respectively. The third IMF (C3) shows that 1.8-hour oscillation is present both during periods of large and small diurnal inequality. Maximum amplitude reached 0.05 feet, making it the smallest oscillation event. A zoom view of C3 and of its corresponding instantaneous frequency (IF) are shown in Figure 16. Most of the IF's values are concentrated between 12 CPD and 15 CPD. The Hilbert Energy Spectrum (HES) shows that the semidiurnal tide is more energetic than the diurnal tide, in particular before YD 296 (Figure 17, Top). The diurnal and semidiurnal tides are equally energetic just after YD 300. From the HES we obtained the Marginal Spectrum (MS), a significant peak was revealed at a frequency of 13 CPD (Figure 17, Bottom). This frequency corresponds to a 1.8-hour oscillation. The diurnal (D) and semidiurnal (SD) peaks show similar energy densities values. The energy density of the 1.8-hour oscillation is about a 100 times lower than the diurnal and semidiurnal tides energy densities.

Discussion

Our results demonstrated the occurrence of a small 1.8-hour oscillation in the sea level records from Isabel Segunda Harbor and Puerto de Fajardo. This oscillation period is shorter than the twelfth-diurnal tidal constituents – with periods from 2.03 to 2.09 hours (see Table 1) – and larger than the sixteen-diurnal harmonic ($8M_2$ or M_{16}; period of 1.55 hours). In addition, it is an intermittent signal in the water-level record. The duration of the larger oscillations is less than 0.5 days. Maximum amplitudes (heights) reached up to 0.2 feet (0.4 feet). The existence of the 1.8-hour oscillation could be related to a harbor oscillation. Harbor oscillations (coastal seiches) are a specific type of seiche motion that occur in partially enclosed basins (gulfs, bays, fjords, inlets, ports, and harbors) that are connected through one or more openings to the sea (Rabinovich, 2009). Long waves entering through the open boundary (harbor entrance) from the open sea are responsible for generating the harbor oscillations. In addition, the harbor seiche losses energy when radiates it through the mouth of the harbor. The fundamental mode of the harbor oscillation is called the Helmholtz mode. The periods of the Helmholtz mode (n=0) and the other harbor modes (n=1, 2, 3…) can be approximately estimated by the following formula (Rabinovich, 2009):

$$T_n = \frac{4L}{(2n+1)\sqrt{gH}}$$

where L is the length of the water body and H is the depth. Both are expressed in meters. The periods are expressed in seconds.

The following table details the period of the harbor oscillation for each natural mode. The periods are expressed in days. We used two sets of length and depth values. The first set (L= 50.77 km, H= 25), represents the length of the platform from Puerto de Fajardo to the shelf break and its average depth. The second set (L= 28.56 km, H=10 m), represents the extension of the shallow shelf that connects Puerto de Fajardo and Isabel Segunda, and its average depth. The first row shows the period of the Helmholtz mode.

Merian's Formula for the periods (natural) of a rectangular basin with uniform depth

Open-ended basin		Open-ended basin	
L (m)	H (m)	L (m)	H (m)
50777	25	28560	10
n	T_n (hours)	n	T_n (hours)
0	3.6	0	3.2
1	1.2	1	1.1
2	0.7	2	0.6
3	0.5	3	0.5
4	0.4	4	0.4

For the first set and second set the Helmholtz period is 3.6 hours and 3.2 hours, respectively. The half of each period is 1.8 hours and 1.6 hours, respectively. The 1.8-hour oscillation has a period equal to half the Helmholtz period. In other words, the period of the 1.8-hour oscillation represents half the fundamental period of oscillation. This result suggests that the period of the 1.8-hour oscillation is determined by the basin geometry and depth.

It is important to emphasize that the above discussion is an oversimplification of the real problem. The above formula applies strictly to a long and narrow harbor, so the values of the periods are approximations to the real values.

References

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Huang, N. E., Shen Z. and S. R. Long. 1999. A new view of nonlinear water waves: the Hilbert Spectrum, Annual Review of Fluid Dynamics 31:417-457.

Huang, N. E., Wu, Z., Long, S. R., Arnold, K. C., Chen, X., Blank, K., 2009. On Instantaneous Frequency. Advances in Adaptive Data Analysis, Vol.1, No. 2, 177-229.

Rabinovich A. B., 2009. Seiches and Harbor Oscillations. Ch. 9. Handbook of Coastal and Ocean Engineering, edited by Y. C. Kim, World Scientific Publ., Singapore, 2009.

Table 1. Twelfth-diurnal Tidal Constituents.

Name	Speed (Degrees/Hour)	Hours	Minutes
5M2NS12	171.800	2.0955	125.7
3(MN)12	172.272	2.0897	125.4
6MNS12	172.344	2.0888	125.3
4M2N12	172.816	2.0831	125.0
7MS12	172.889	2.0823	124.9
4M2NKS12	172.898	2.0822	124.9
5MSNK12	173.278	2.0776	124.7
3N2MS12	173.362	2.0766	124.6
5MN12	173.360	2.0766	124.6
5Mnu12(5Mν12)	173.433	2.0757	124.5
6MSK12	173.822	2.0711	124.3
3M2SN12	173.832	2.0710	124.3
MA12	173.864	2.0706	124.2
M12	173.905	2.0701	124.2
4MSN12	174.376	2.0645	123.9
4ML12	174.449	2.0636	123.8
4MNK12	174.458	2.0635	123.8
2(MSN)12	174.848	2.0589	123.5
5MT12	174.879	2.0586	123.5
5MS12	174.921	2.0581	123.5
5MK12	175.003	2.0571	123.4
3M2SN12	175.392	2.0525	123.2
6MSN12	175.465	2.0517	123.1
3MNKS12	175.474	2.0516	123.1
5MSN12	175.547	2.0507	123.0
4MST12	175.895	2.0467	122.8
4M2S12	175.936	2.0462	122.8
4MSK12	176.019	2.0452	122.7
3(MS)12	176.952	2.0344	122.1
3M2SK12	177.034	2.0335	122.0

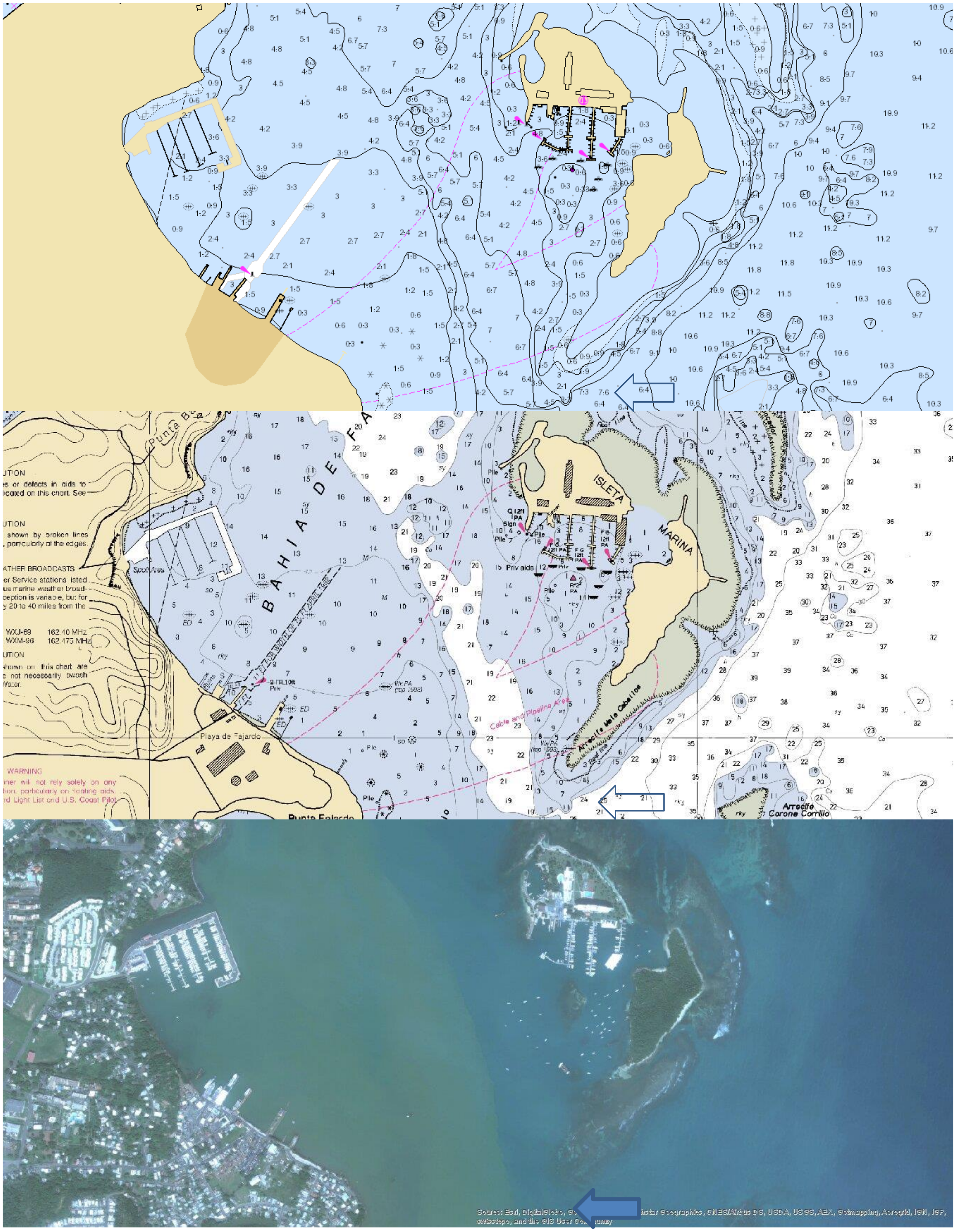


Figure 1. Puerto de Fajardo shown in two charts and a satellite image.

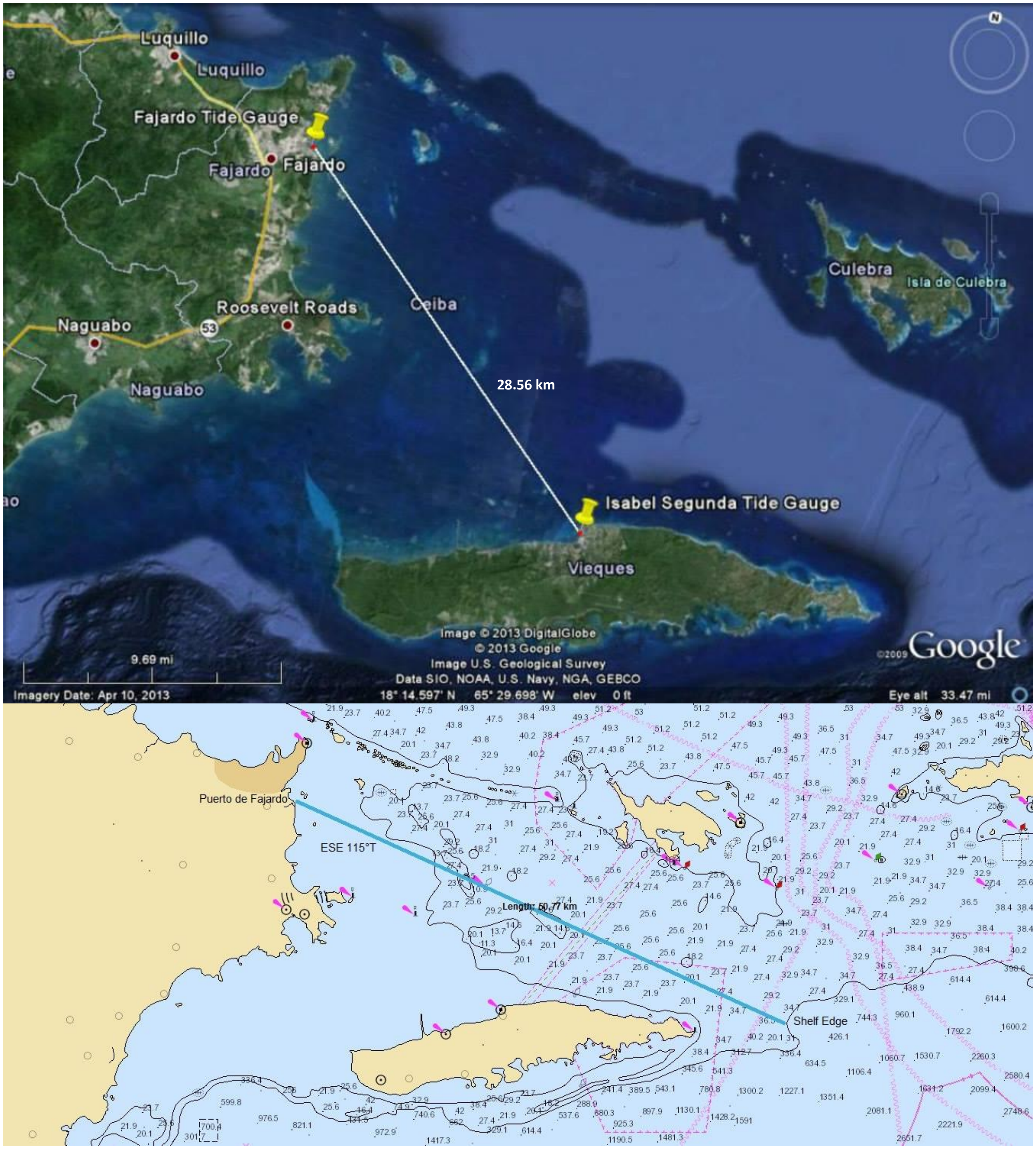


Figure 2. (Top) Distance between the two harbors. (Bottom) A nautical chart shows the depth soundings in meters and the distance to the shelf break in kilometers.

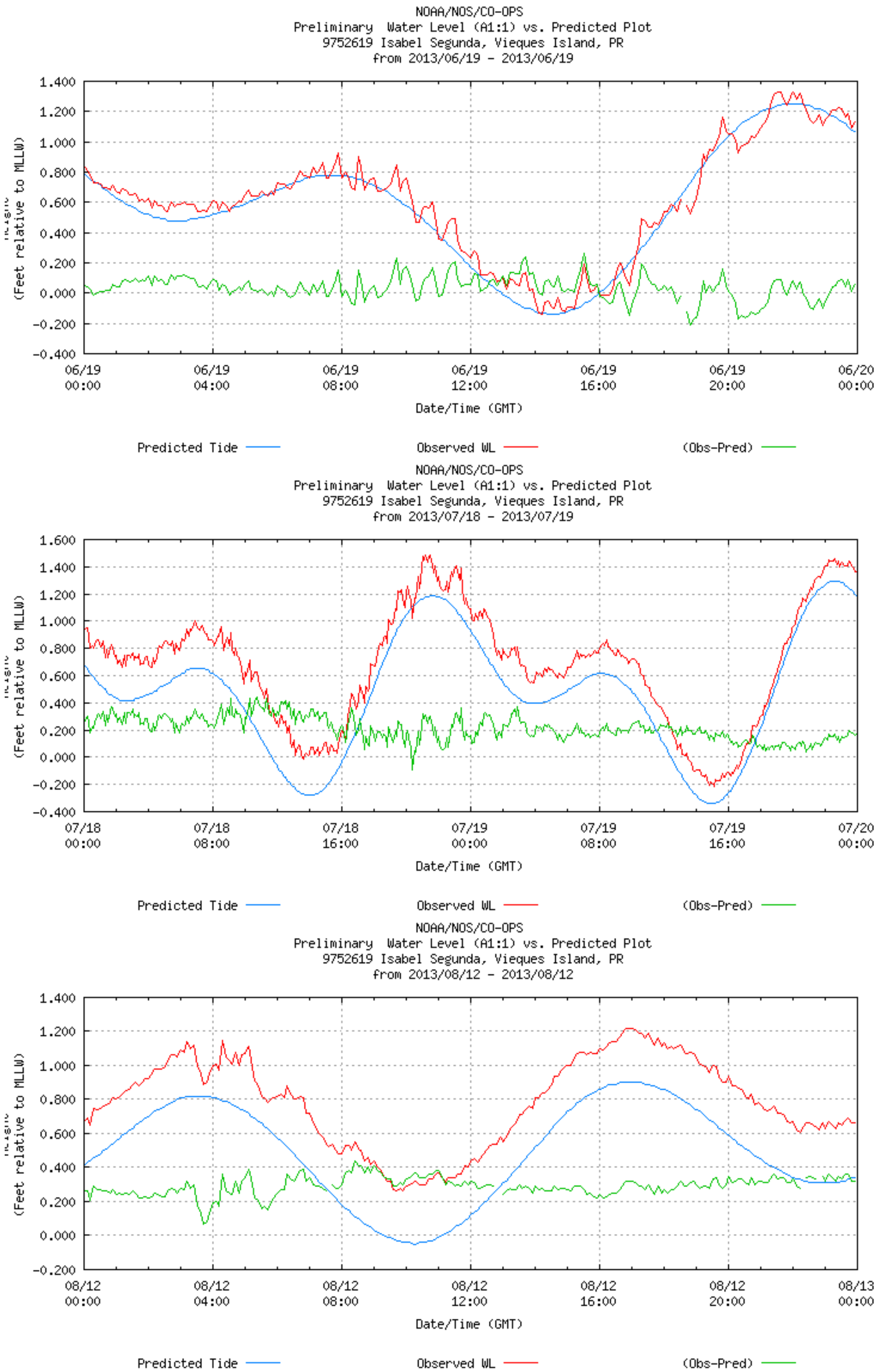
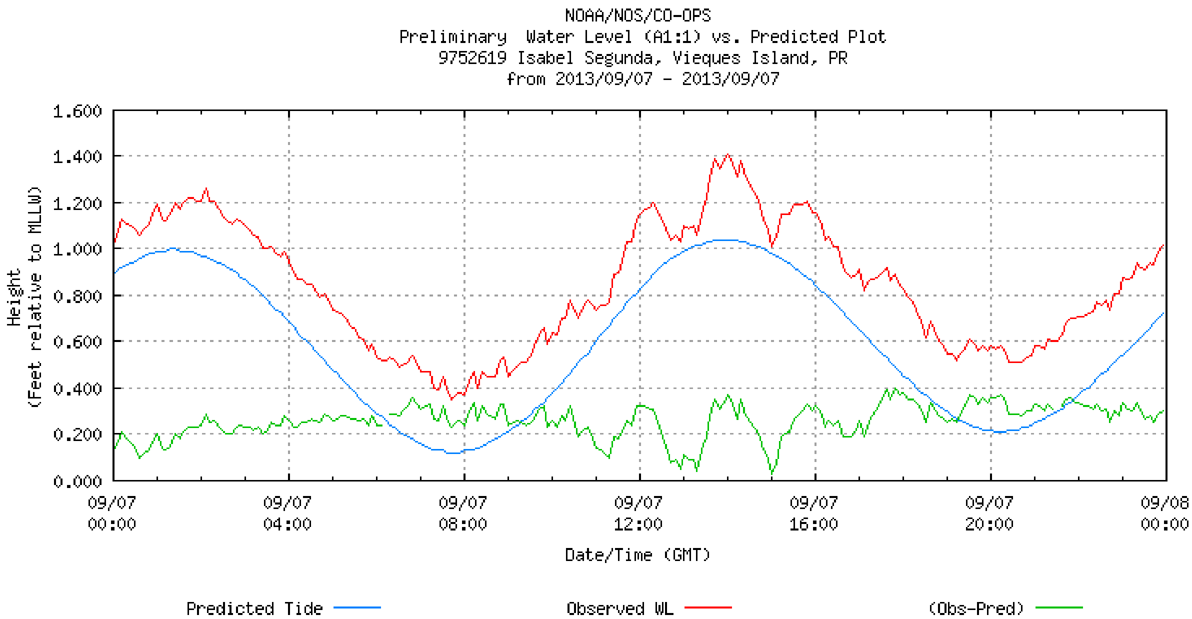


Figure 3. Water level records from Isabel Segunda Tidal Station for three events on June 19, July 19 and August 12 2013.



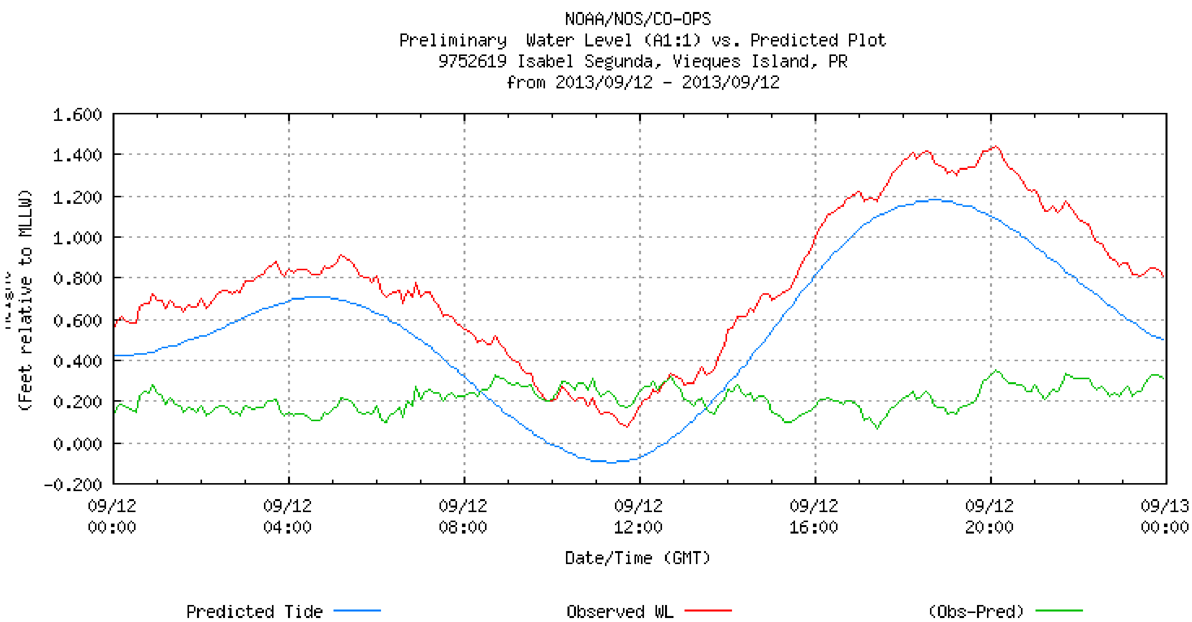
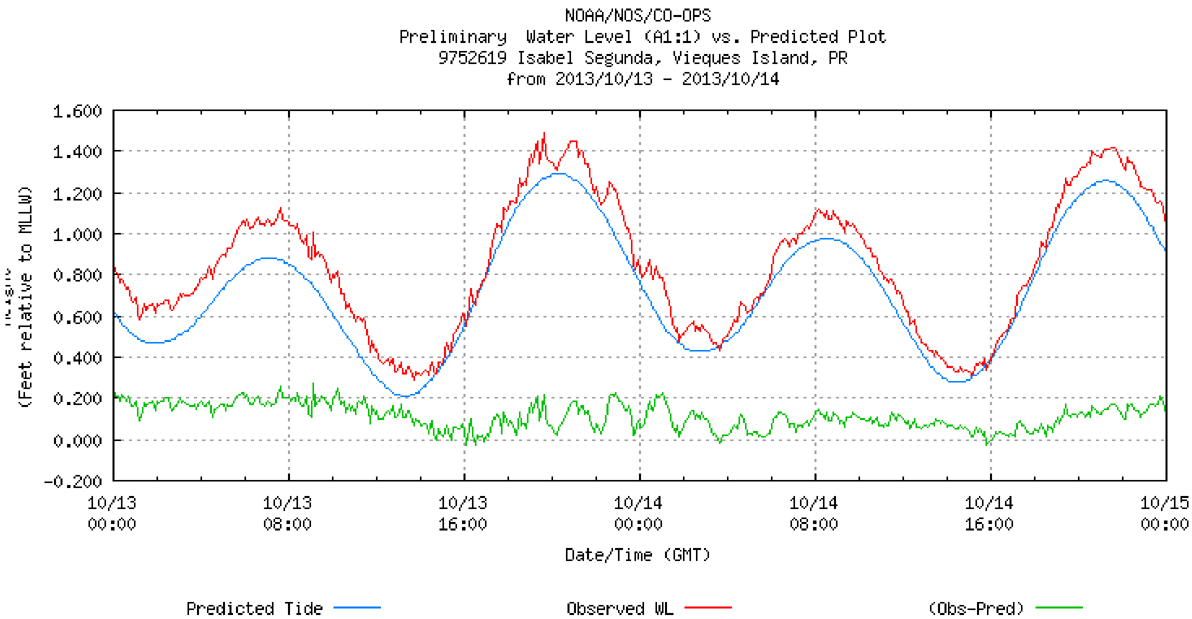


Figure 4. Water level records from Isabel Segunda Tidal Station showing two events on September 7 and September 12 2013.



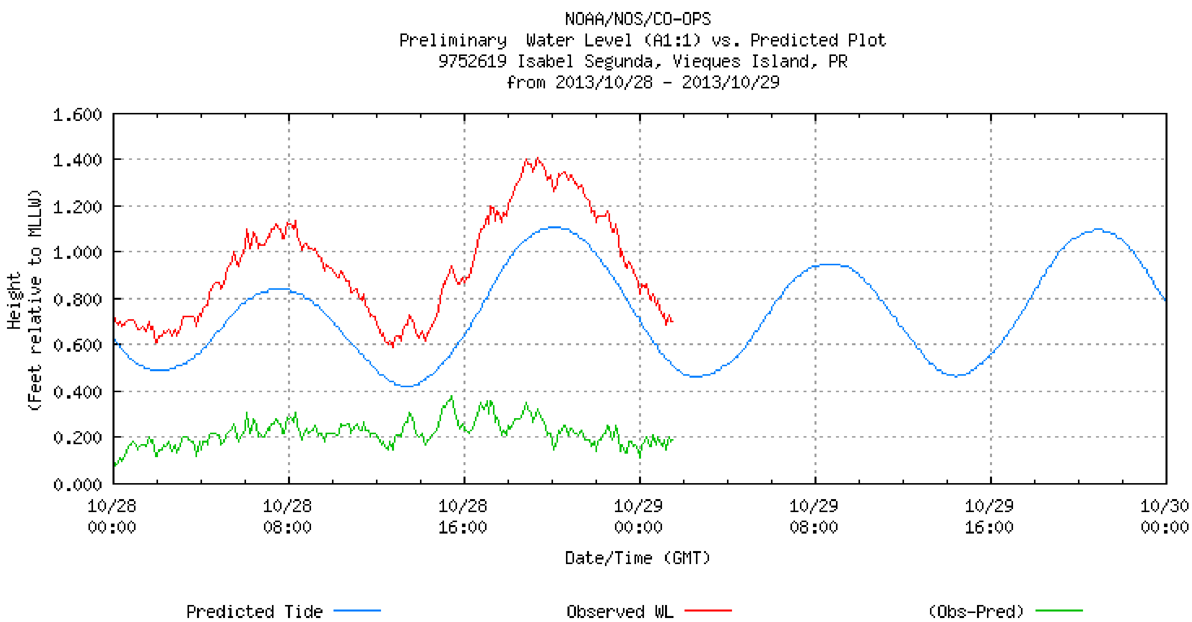
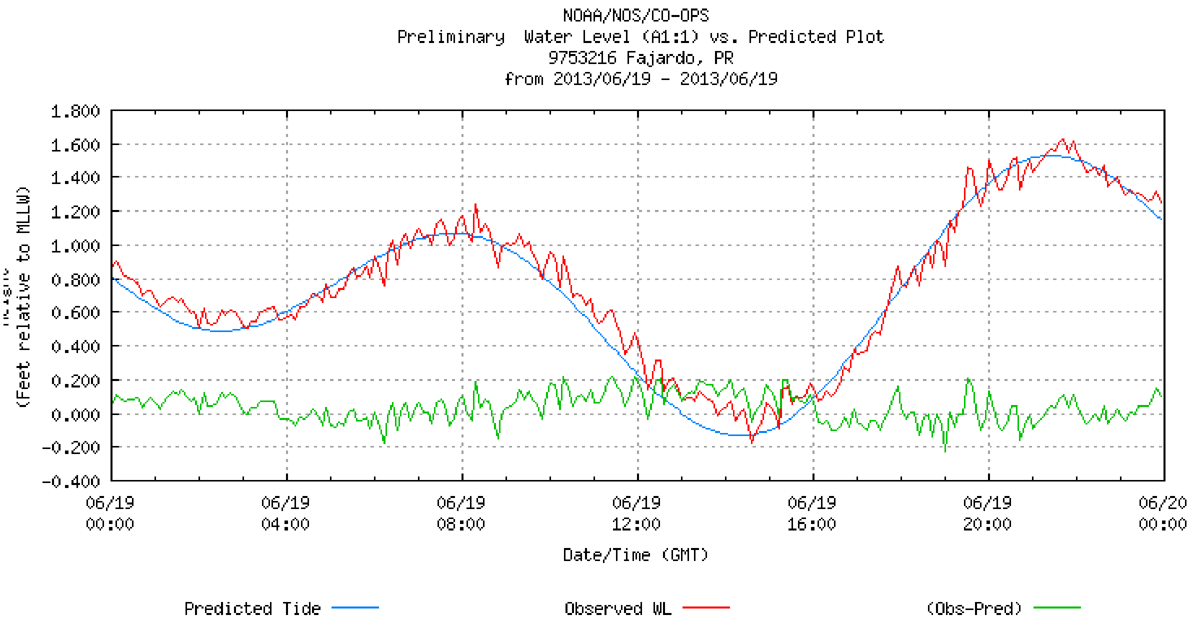


Figure 5. Water level records from Isabel Segunda Tidal Station showing two events on October 13-14 and October 28 2013.



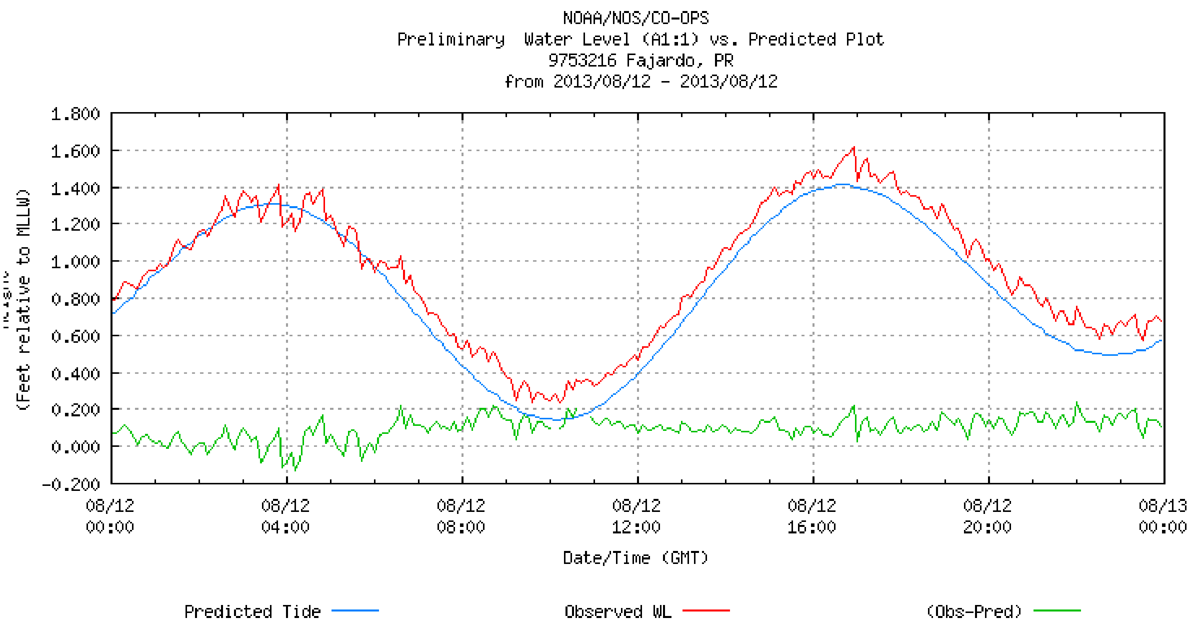


Figure 6. Water level records from Puerto de Fajardo Tidal Station showing two events on June 19 and August 12 2013.

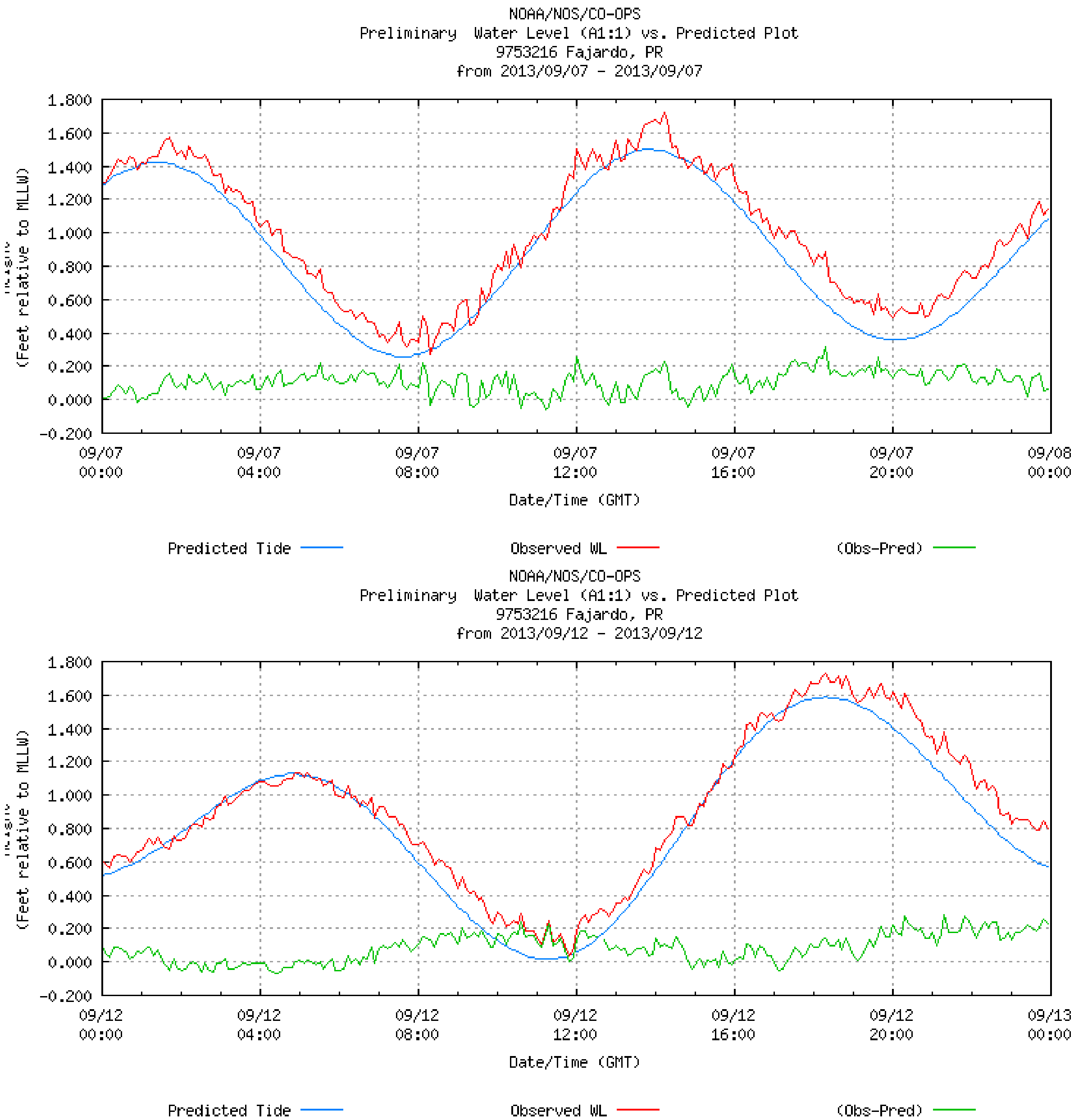


Figure 7. Water level records from Puerto de Fajardo Tidal Station showing two events on September 7 and September 12 2013.

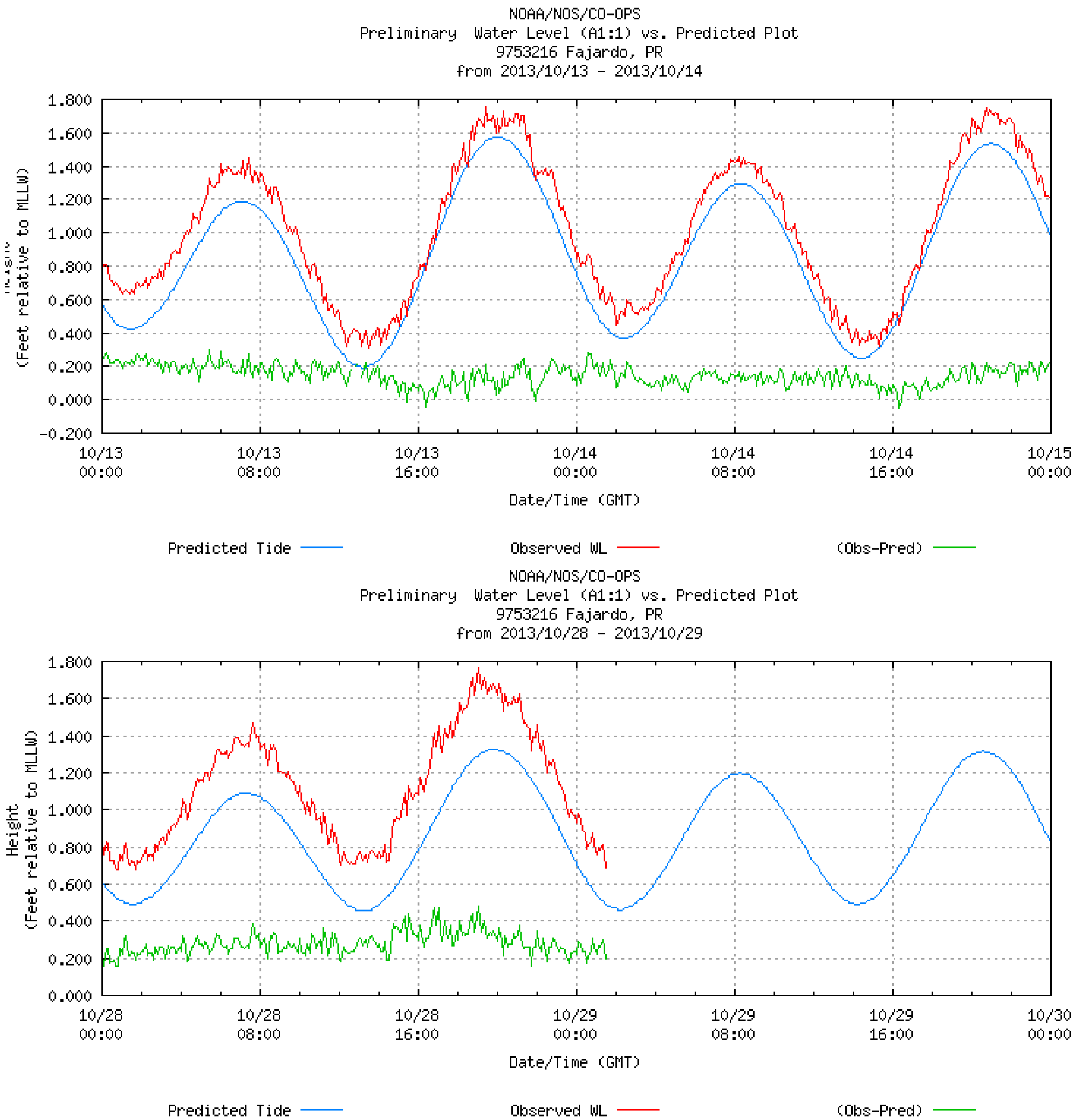


Figure 8. Water level records from Puerto de Fajardo Tidal Station showing two events on October 13-14 and October 28 2013.

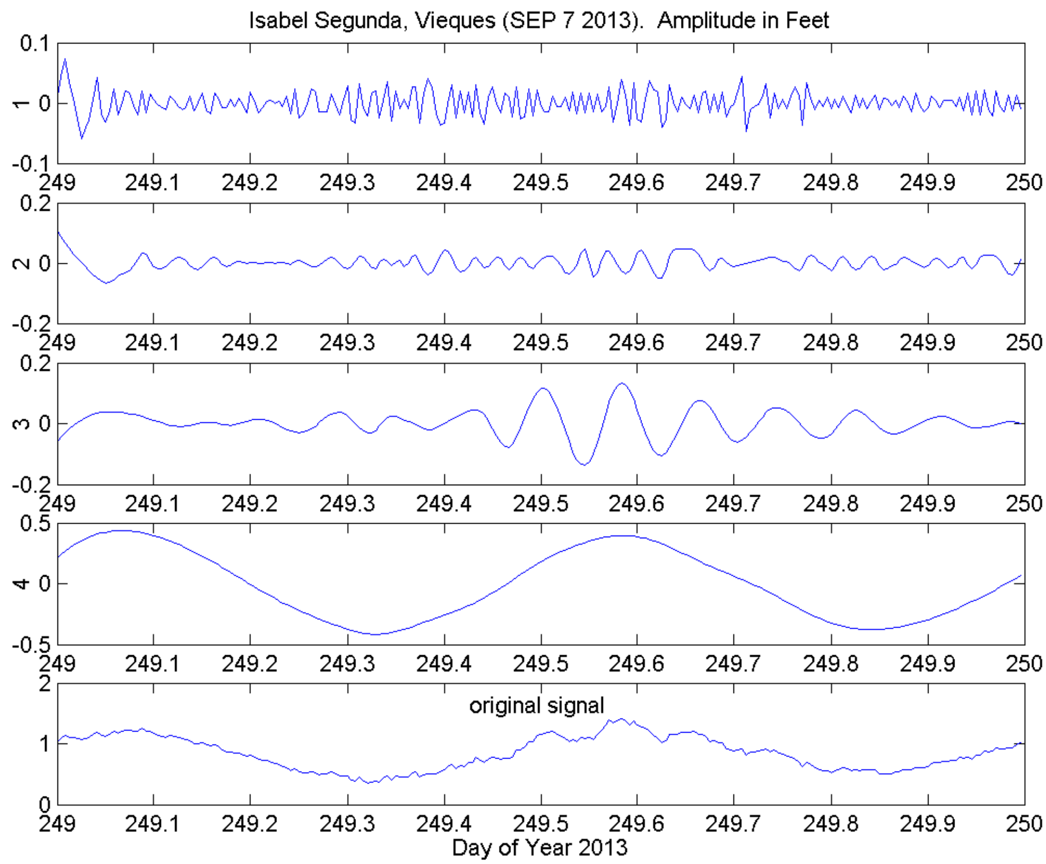


Figure 9. EMD analysis of sea-level from Isabel Segunda on September 7, 2013. The third panel shows the IMF representing the 1.9 hour oscillation (C3). The fourth panel is the semidiurnal tide and the bottom panel is the original record. Amplitudes are expressed in feet.

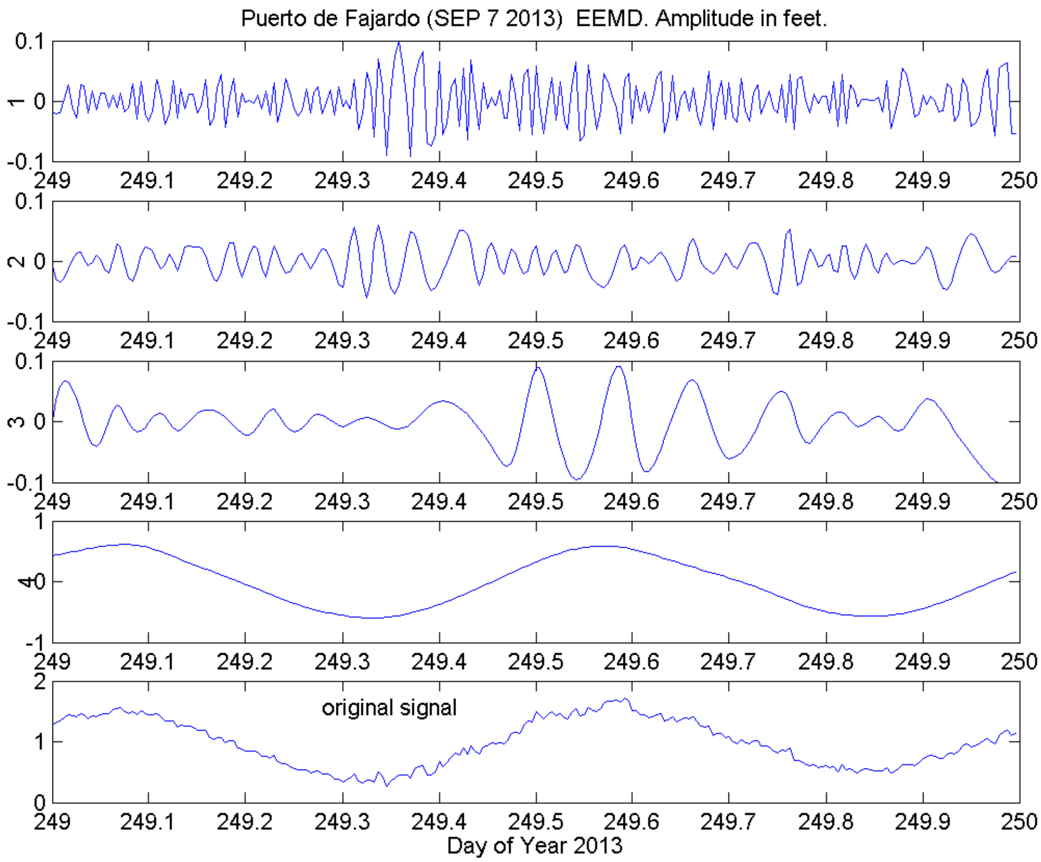


Figure 10. EMD analysis of sea-level from Puerto de Fajardo on September 7, 2013. The third panel shows the IMF representing the 1.9 hour oscillation (C3). The fourth panel is the semidiurnal tide and the bottom panel is the original record. Amplitudes are expressed in feet.

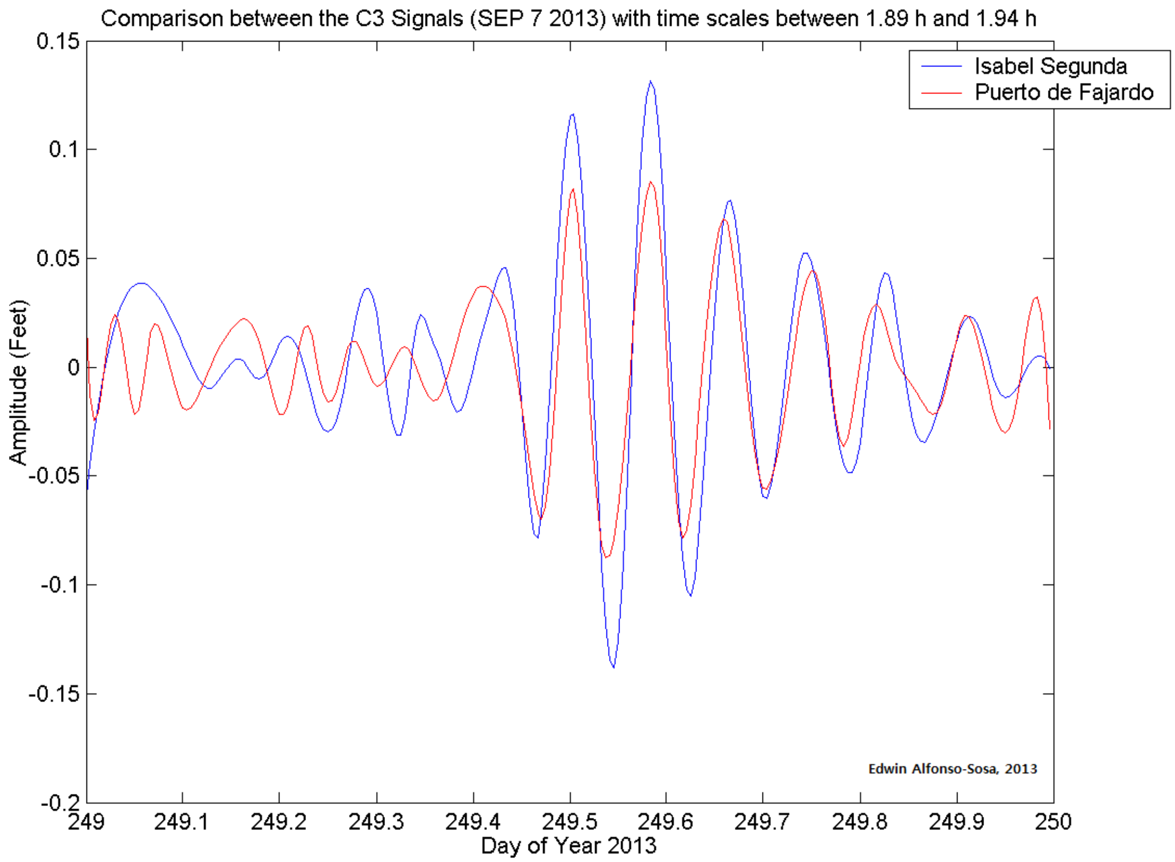


Figure 11. Overlay of the third IMF components (C3) from the EMD analysis of the water level records at Isabel Segunda and Puerto de Fajardo on September 7 2013.

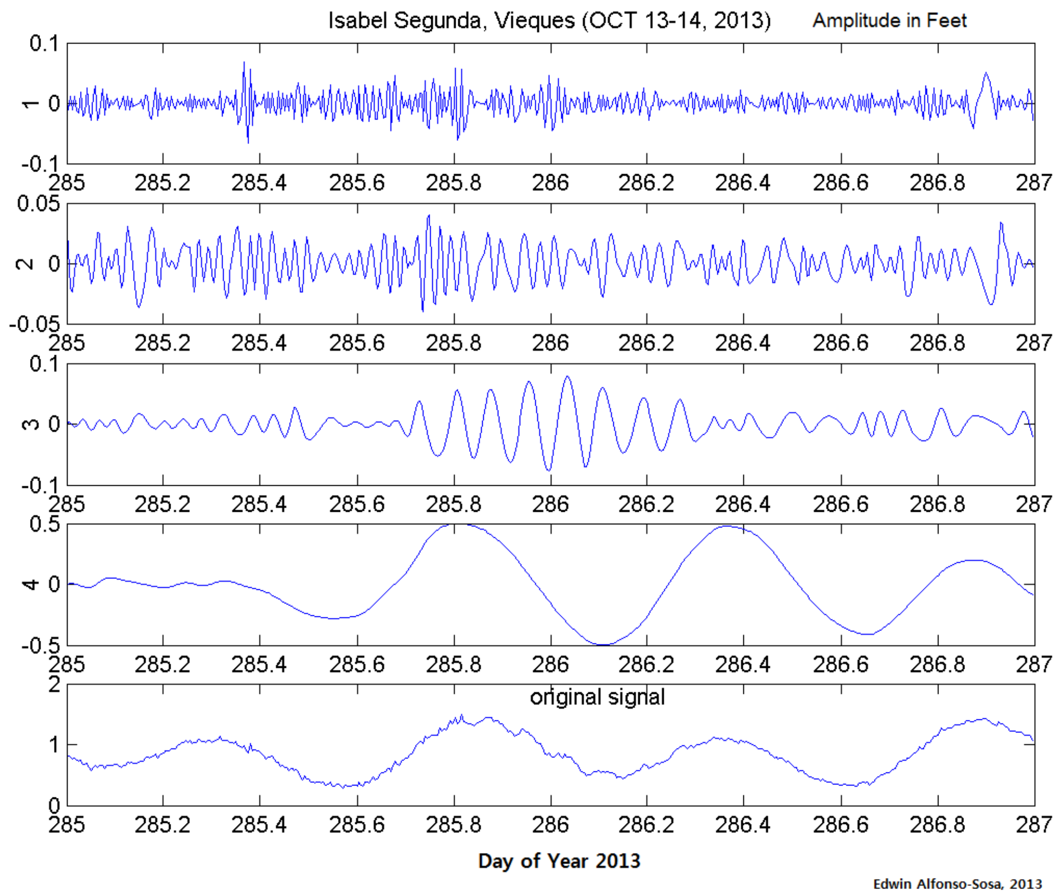


Figure 12. EMD analysis of sea-level from Isabel Segunda on October 13-14, 2013. The third panel shows the IMF representing the 1.8 hour oscillation (C3). The fourth panel is the semidiurnal tide and the bottom panel is the original record. Amplitudes are expressed in feet.

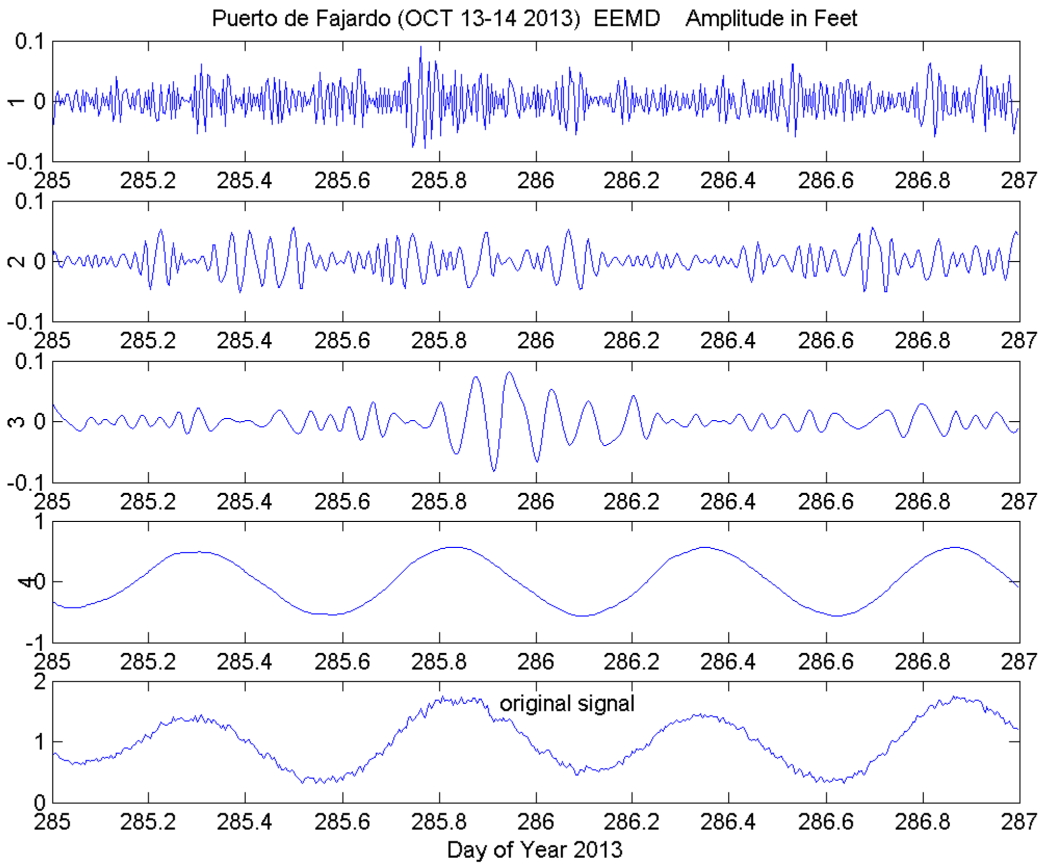


Figure 13. EMD analysis of sea-level from Puerto de Fajardo on October 13-14, 2013. The third panel shows the IMF representing the 1.8 hour oscillation (C3). The fourth panel is the semidiurnal tide and the bottom panel is the original record. Amplitudes are expressed in feet.

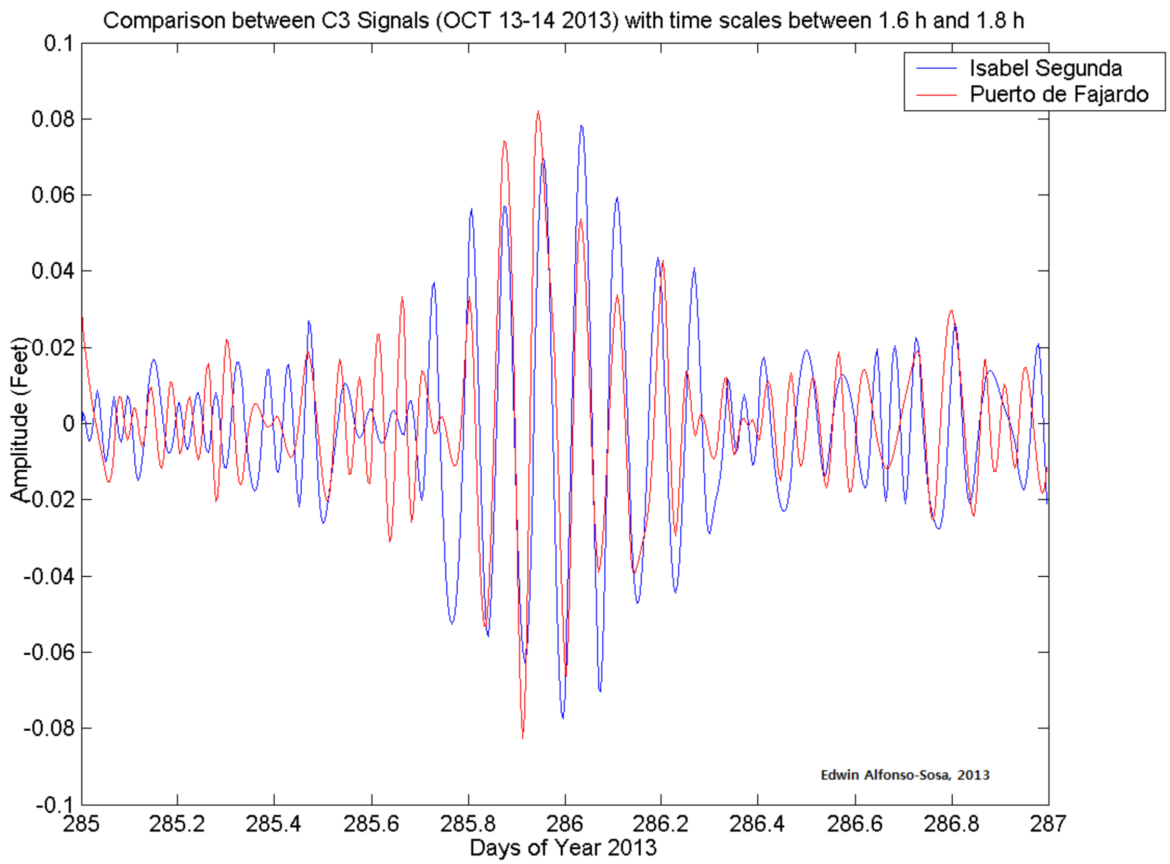


Figure 14. Overlay of the third IMF components (C3) from the EMD analysis of the water level records at Isabel Segunda and Puerto de Fajardo on October 13-14 2013.

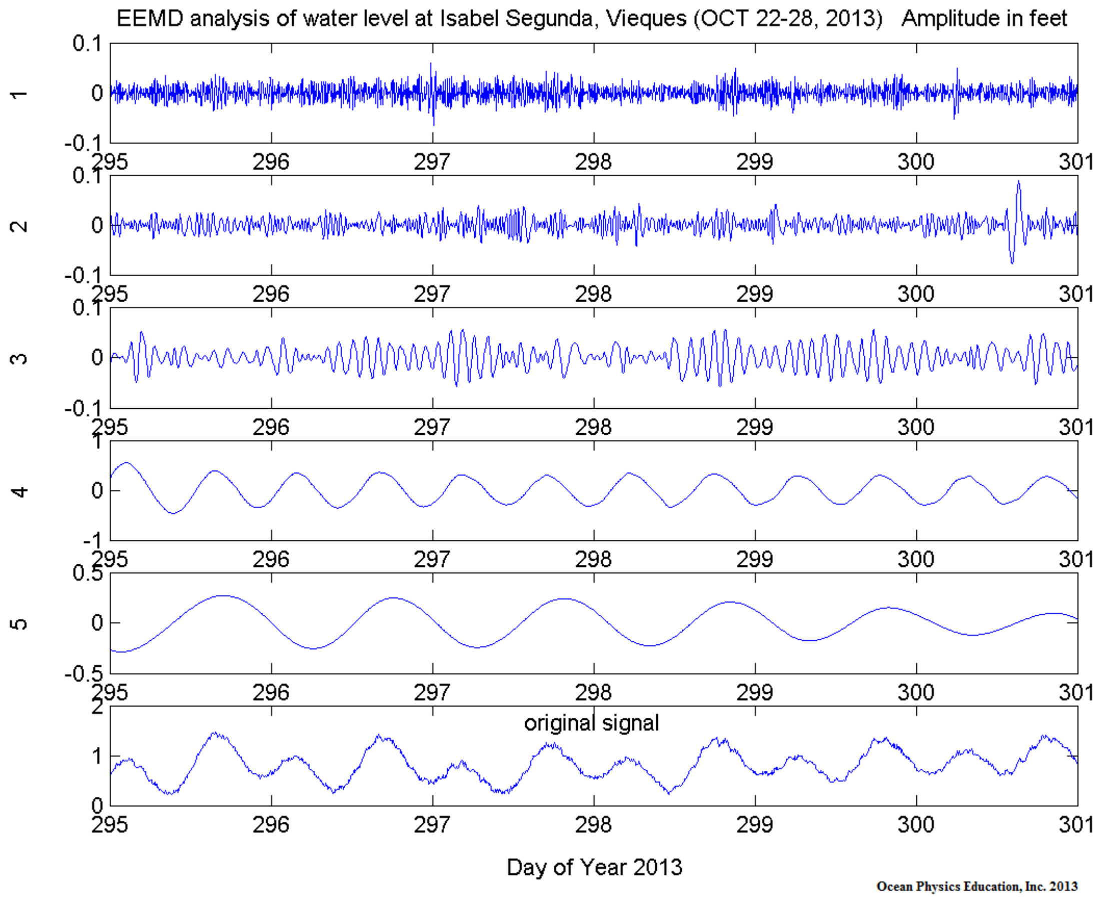


Figure 15. EMD analysis of sea-level from Isabel Segunda on October 22-28, 2013. The third panel shows the IMF representing the 1.84 hour oscillation (C3). The fourth panel is the semidiurnal tide, the fifth is the diurnal tide and the bottom panel is the original record. Amplitudes are expressed in feet.

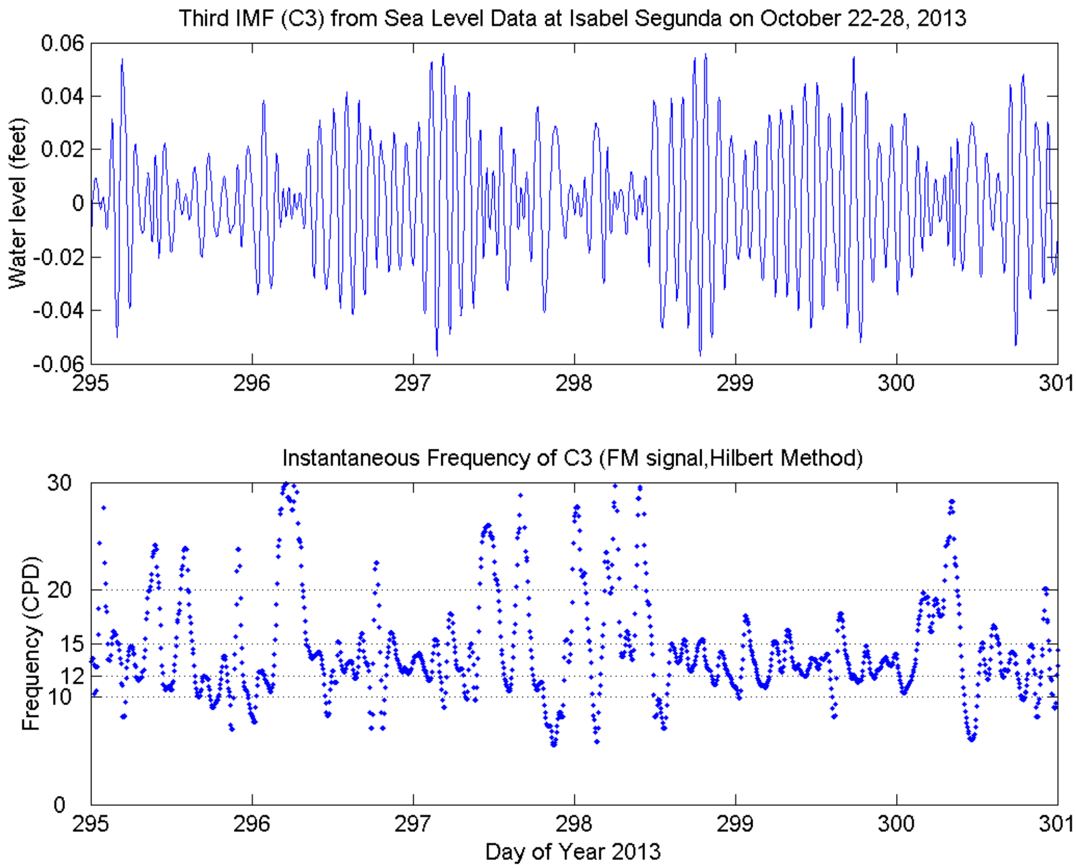


Figure 16. (Top) The third IMF component (C3) from the EMD analysis of the water level records at Isabel Segunda on October 22-28, 2013. (Bottom) The instantaneous frequency of C3 modulates between 10 and 20 cycles per day (CPD). Note that during periods of larger amplitudes of C3, between YD 296.5-297.3 and YD 298.6-YD 300.0, the instantaneous frequency values ranged between 12 CPD and 15 CPD.

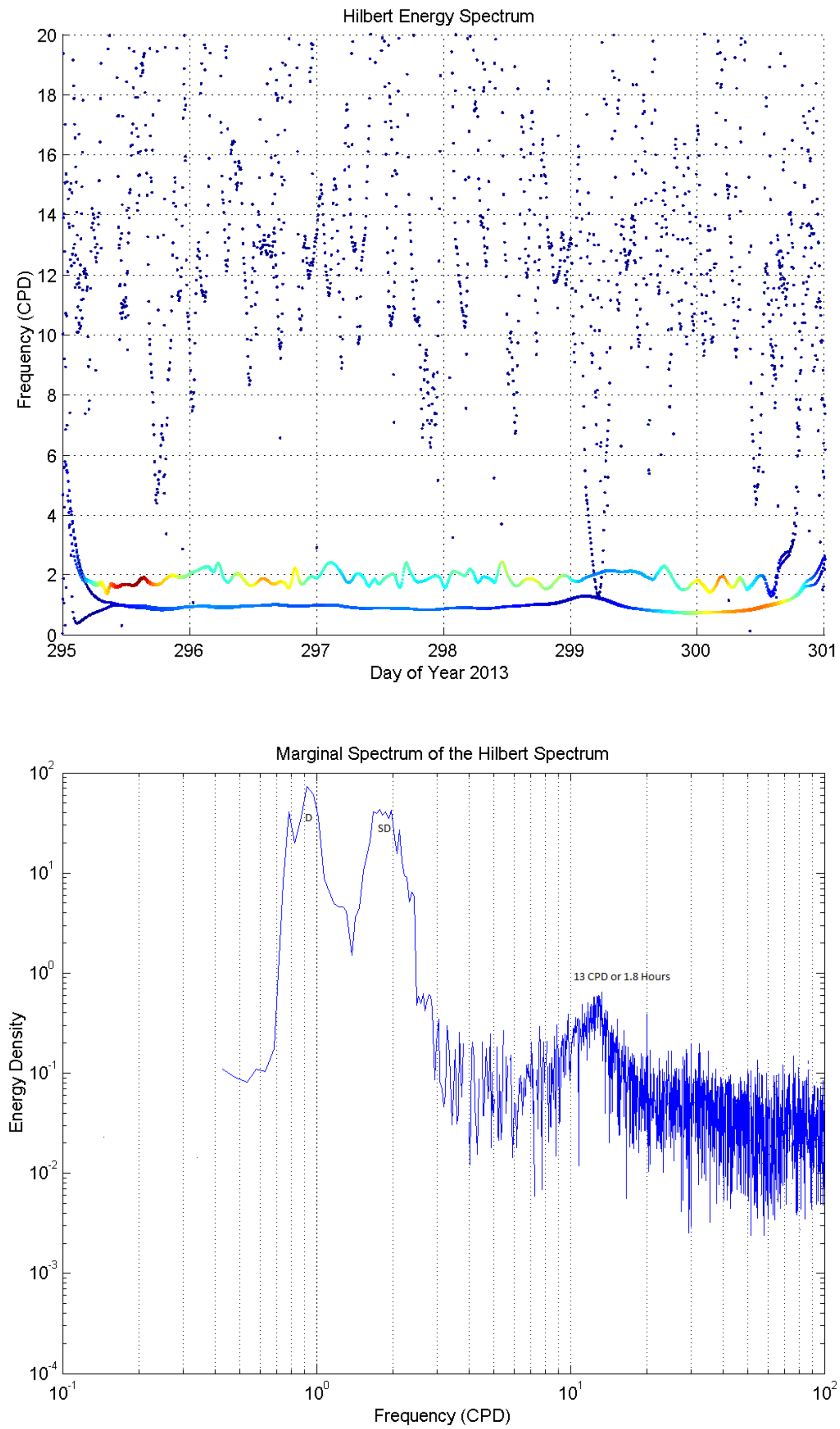


Figure 17. (Top) Hilbert Energy Spectrum of the sea level at Isabel Segunda and (Bottom) the Marginal Spectrum showing clearly the 1.8-hour oscillation, the semidiurnal tide (SD) and the diurnal tide (D).